A memetic algorithm with adaptive operator selection for graph coloring

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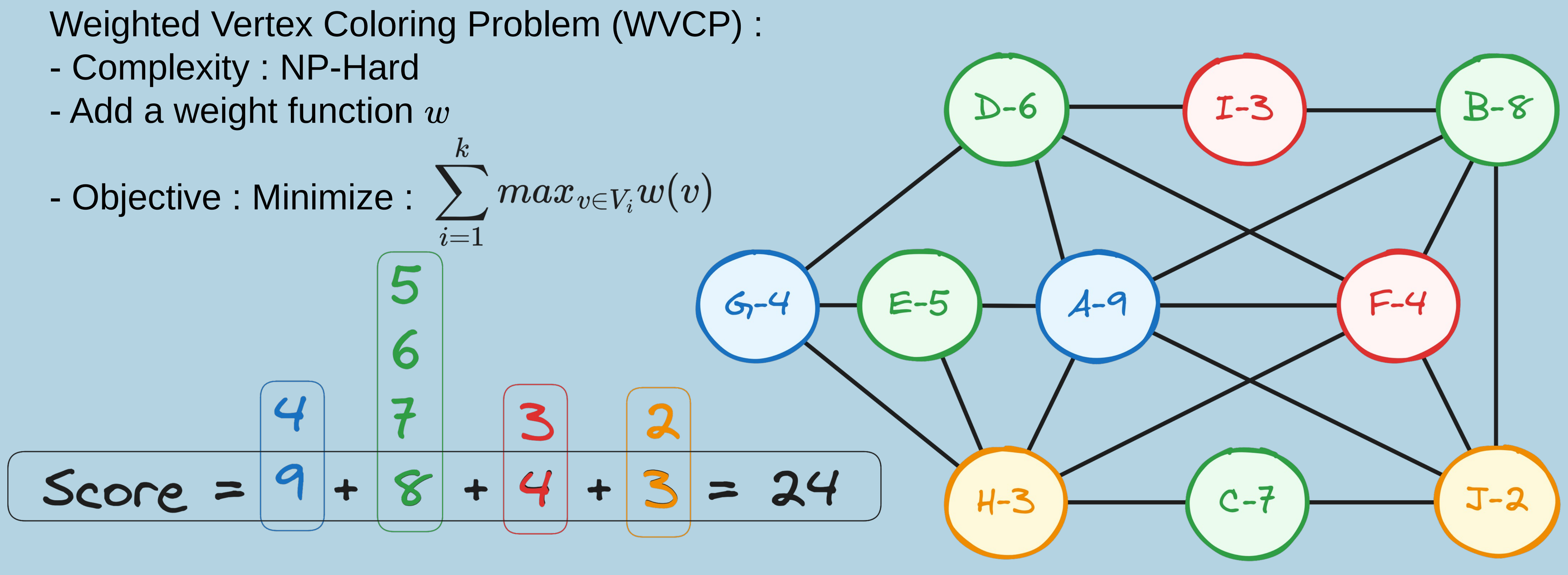
Studied Problems

An undirected graph instance is defined by G=(V,E), V the set of vertices, E the set of edges.

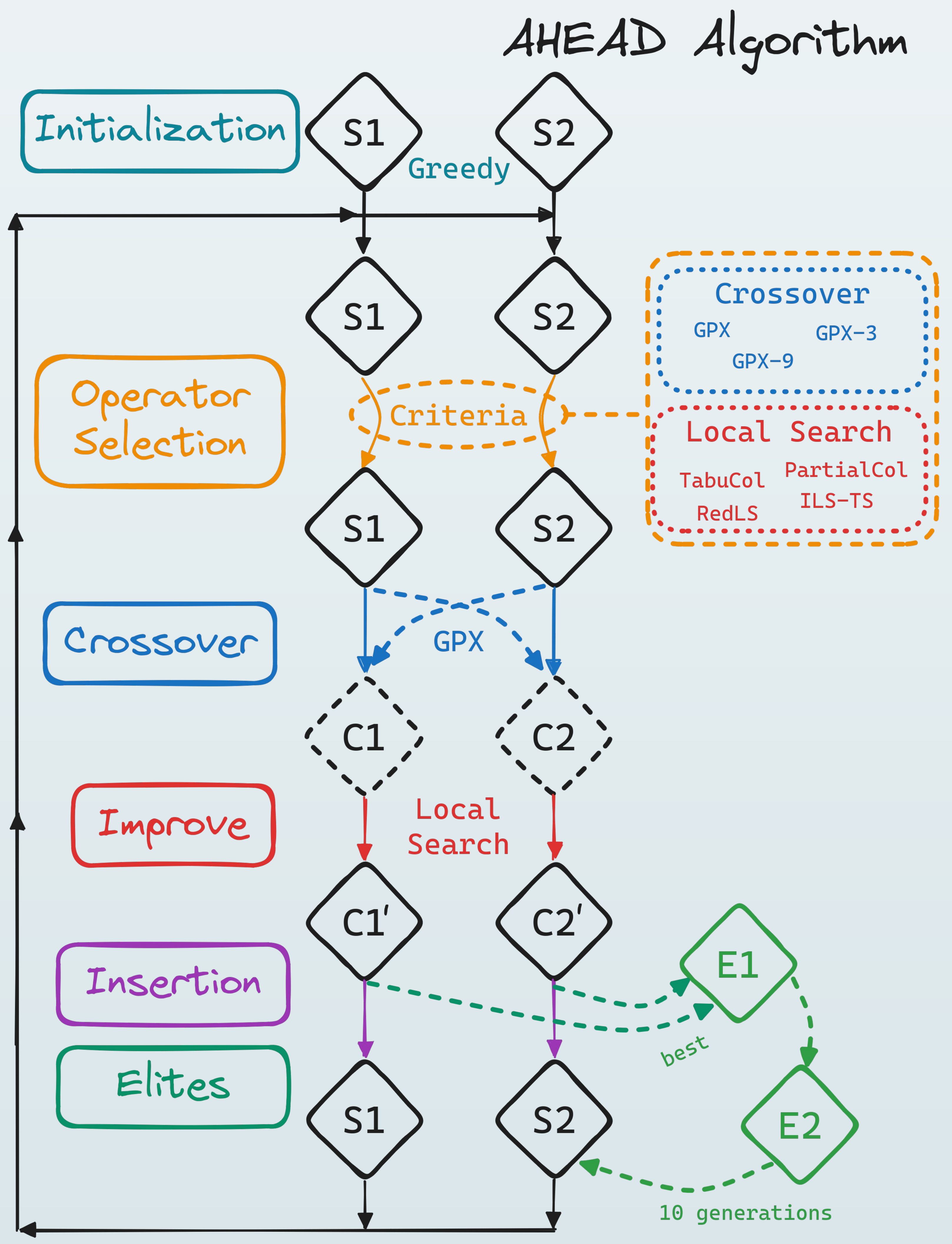
Graph Coloring Problem (GCP) :
- Complexity : NP-Hard
- Objective : find a legal coloring that minimizes the number of colors

Weighted Vertex Coloring Problem (WVCP) :
- Complexity : NP-Hard
- Add a weight function w
- Objective : Minimize : $\sum_{i=1}^{k} max_{v \in V_i} w(v)$



Memetic Algorithm

Based on HEAD [Moalic & Gondran, 2018] (Hybrid Evolutionary Algorithm in Duet), we propose AHEAD (Adaptive HEAD). A two individuals memetic algorithm with an adaptive selection of crossover and local search algorithm. The objective of the algorithm is to learn during the search which pair of crossover and local search to use on the current instance.



Possible Operators

Local Search :

GCP :

- **TabuCol** [Hertz & Werra, 1987]
- **PartialCol** [Blöchliger & Zufferey, 2008]

WVCP :

- **RedLS** [Wang et al., 2020]
- **ILS-TS** [Nogueira et al., 2021]

Crossover :

3 variations of the GPX crossover operator (HEA [Galinier & Hao, 1999])

GPX : alternatively select the largest color in each parent P1 and P2

GPX-3 : 3 colors in P1 for 1 color in P2

GPX-9 : 9 colors in P1 for 1 color in P2

Selection Criteria

6 criteria are proposed for selecting pairs of operators :
- **Random** : Uniform random choice
- **Deleter** : Delete the least performing operators (o)
- **Roulette** : Random selection weighted by rewards (r)

$$proba[o] = p_{min} + (1 - |O| * p_{min}) * \frac{r[o]}{\sum r}$$

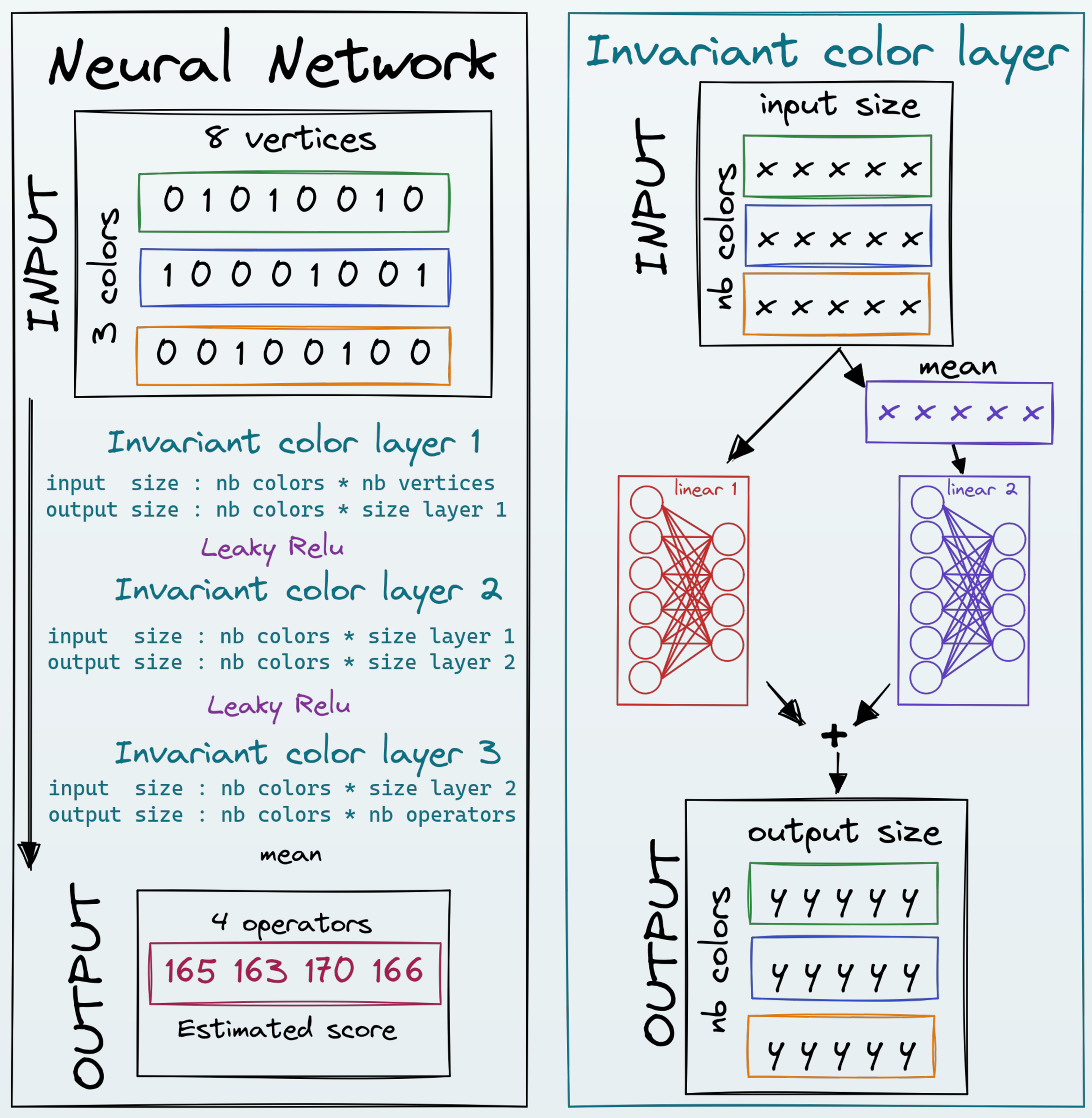
- **Pursuit** : Selection in favor of the best operator (b)

$$\begin{cases} proba[b] = proba[b] + \beta(p\_max - proba[b]) \\ proba[o] = proba[o] + \beta(p\_min - proba[o]) \end{cases}$$

- **UCB** : Focusing on the best while encouraging exploration

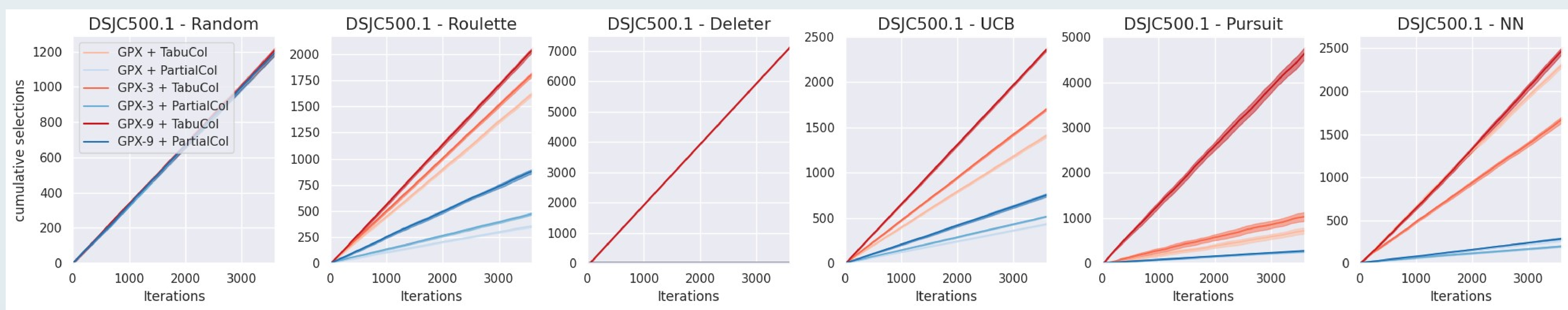
$$score[o] = r[o] + c * \sqrt{2 * \frac{\log(\sum visits)}{visits[o]}}$$

- **NN** : Recommendation of a neural network on a raw solution with Deep Sets



Results – Choice of operators

Plot of the cumulative selection for each operator with the different criteria :



No change in operator choice during search
More conservative GPX crossover slightly preferred
More importance given to local search choice

Results – Comparison to other methods

Tests on the hardest benchmark instances. (BKS = Best Known Score in the literature)
GCP :

/31 instances	PartialCol	TabuCol	HEAD+ Tabucol	AHEAD+Random	AHEAD+Deleter
#BKS	5	8	7	9	**13**
#Best	8	14	17	17	**24**
#Best avg	11	7	15	9	**20**

WVCP :

/48 instances	RedLS	ILS-TS	HEAD+RedLS	AHEAD+Random	AHEAD+Deleter
#BKS	15	23	19	21	**24**
#Best	24	25	19	22	**28**
#Best avg	11	**21**	11	19	19

Conclusion

Among the selection criteria, the most elitists ones obtain the best results, in particular for Deleter but also Pursuit.

On GCP, AHEAD is better than local searches alone, HEAD+TabuCol remain equal in some cases as TabuCol is more often better than PartialCol, overall, AHEAD+Deleter obtain better results.

On the WVCP, the local searches stay significantly better on few instances (9 for RedLS, less than 6 for ILS-TS) but AHEAD+Deleter and Pursuit obtain good results.

AHEAD is able to find new best scores :
for the GCP : C2000.9 (404)
for the WVCP : le450_15a (211) and le450_15b (215) and queen14_14 (214)