A memetic algorithm with adaptive operator selection for graph coloring

AHEAD (Adaptive HEAD) Algorithm

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EVOCOP 2024

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Studied Problems

GCP - Graph Coloring Problem

Graph Coloring

Objective: find a legal color that minimizes the number of colors

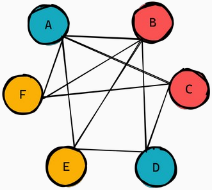
Score :

- Number of colors k (legal)
- Number of conflicts $|C|$ (illegal)
- Number of uncolored vertices $|U|$ (partially legal)

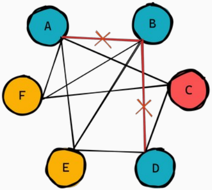
NP-Hard problem

Applications :

- Scheduling problems
- Register allocation
- Sudoku
- . . .



legal coloring
with 3 colors



Unsatisfied constraints
2 conflicts

WVCP - Weighted Vertex Coloring Problem

Weighted Vertex Coloring

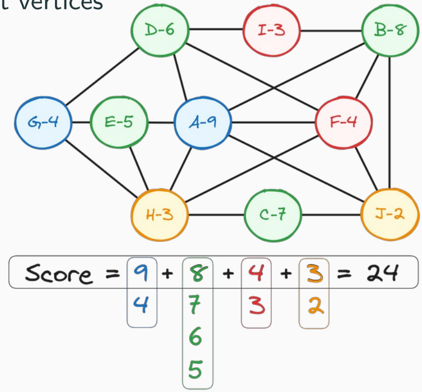
Objective: find a legal coloring that minimizes the sum of the weights of the heaviest vertices in each color

Score : $\sum_{i=1}^{k} max_{v \in V_i} w(v)$

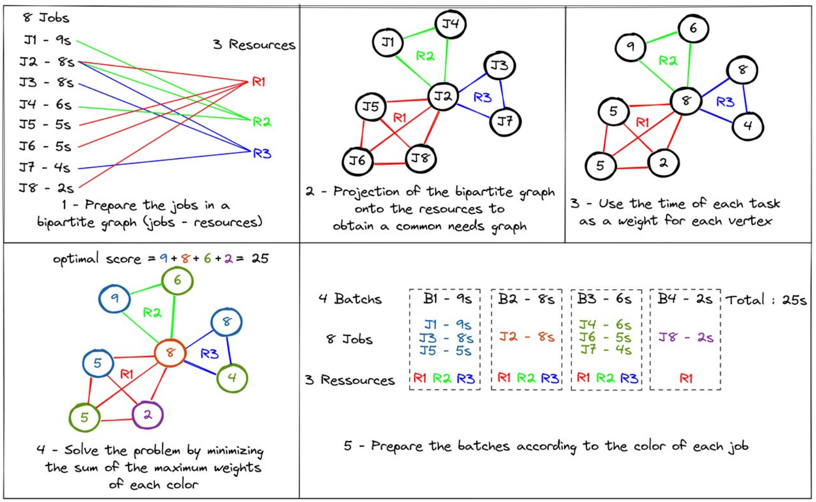
NP-Hard problem

Applications :

- Traffic management in satellite communications
- Matrix decomposition problem
- Scheduling batch job in parallel



WVCP - Scheduling Parallel Batch Jobs



State of the Art

GCP - State of the Art

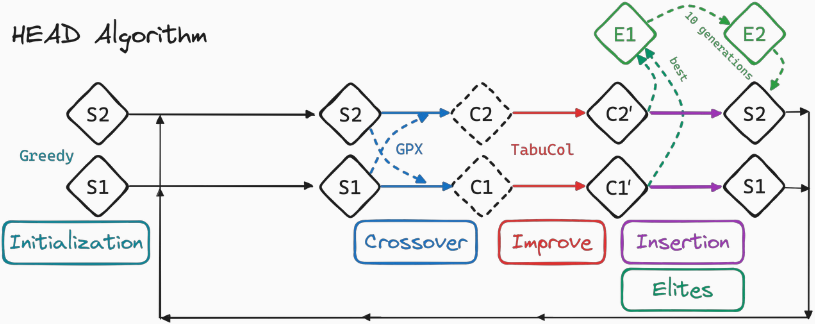
- Local Search:
  - **TabuCol** Hertz et Werra [1987] : illegal, one-move
  - **PartialCol** Blöchliger et Zufferey [2008] : partial legal, grenade
  - **ILS** Chiarandini et Stützle [2002] : perturbations, acceptance criteria
- Memetic Algorithms :
  - **HEA** Galinier et Hao [1999] : GPX, TabuCol
  - **Evo-Div** Porumbel *et al.* [2010] : multi-parents crossover, distances
  - **MACOL** Lü et Hao [2010] : multi-parents crossover, distances
  - **HEAD** Moalic et Gondran [2018] : 2 individuals, GPX, TabuCol
  - **DLMCOL** Goudet *et al.* [2022] : +20 000, NN select crossover
  - **AHEAD** Grelier *et al.* [2024] : Adaptive HEAD
- Learning :
  - **PLSCOL** Zhou *et al.* [2018] : local search, reinforcement learning
  - **TensCol** Goudet *et al.* [2021] : tensor, gradient descent
  - **NRPA** Cazenave *et al.* [2021] : MCTS, sequence, gradient descent

WVCP - State of the Art

- Learning :
  - **MCTS + Local Search** Grelier *et al.* [2022] LS as simulation
  - **MCTS + Hyperheuristics** Grelier *et al.* [2023] : select LS
- Memetic Algorithms :
  - **DLMCOL** Goudet *et al.* [2022] : +20000, NN select crossover
  - **AHEAD** Grelier *et al.* [2024] : Adaptive HEAD
- Local Search :
  - **AFISA** Sun *et al.* [2018] : illegal, one-move, adaptive coefficient
  - **RedLS** Wang *et al.* [2020] : illegal, weighted edges, perturbations
  - **ILS-TS** Nogueira *et al.* [2021] : p-legal, 6 neighbors, perturbations
  - **TabuWeight** Grelier *et al.* [2022] : legal, one-move
- Exact Methods :
  - **2-Phase** Malaguti *et al.* [2009] : column generation, ILP
  - **MWSS** Cornaz *et al.* [2017] : MIP, max weight stable set problem
  - **CP** Goudet *et al.* [2023] : 3 CP models, reduction

Memetic Algorithm – AHEAD

HEAD - Hybrid Evolutionary Algorithm in Duet



Moalic et Gondran [2018] – Variations on memetic algorithms for graph coloring problems

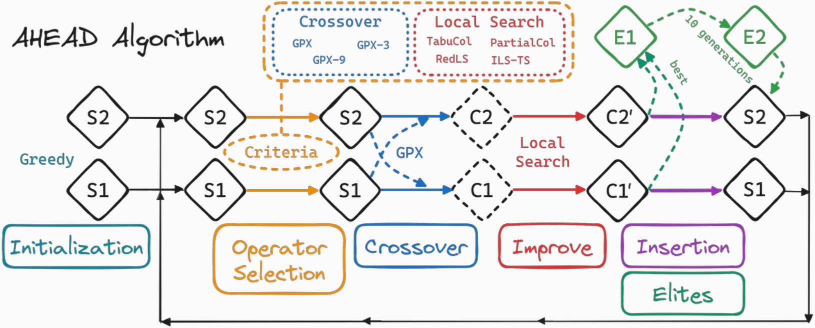
Why HEAD?

- One of the best algorithm for GCP
- Simple and efficient

How?

- 2 individuals
- GPX
- TabuCol

AHEAD - Adaptive Hybrid Evolutionary Algorithm in Duet



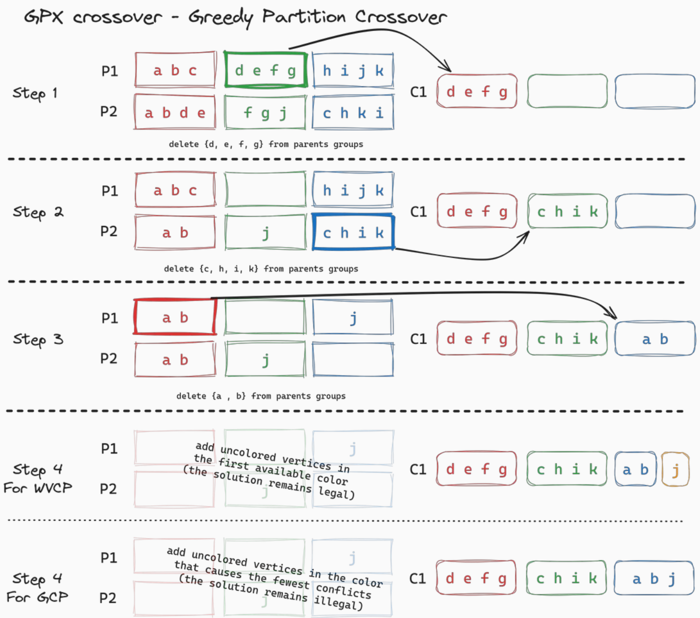
Why AHEAD?

- Attempt to improve HEAD
- Adapt to the instance

How?

- Hyperheuristics
- Multiple crossover
- Multiple local search

GPX - Galinier et Hao [1999]



AHEAD - Operators

Crossover

- **GPX** : 1 color in P1 for 1 color in P2
- **GPX-3** : 3 colors in P1 for 1 color in P2
- **GPX-9** : 9 colors in P1 for 1 color in P2

GCP - Local Search

- **TabuCol** : Hertz et Werra [1987]
- **PartialCol** : Blöchliger et Zufferey [2008]

WVCP - Local Search

- **RedLS** : Wang *et al.* [2020]
- **ILS-TS** : Nogueira *et al.* [2021]

AHEAD – Hyperheuristics

Why?

- Diversification during the search
- No best operator on all instances
- Adapt to the instance

How?

- Selection criteria : Learn to select the best operator
- Reward : Score of the solution after the localsearch
- Selection : pair $<$crossover, local search$>$
- Exception : NN : Generate all crossovers and select the best one

Hyperheuristics - Criteria

Selection criteria

- **Random** Uniform random choice
- **Deleter** Delete the least performing operators (o)
- **Roulette** Goëffon *et al.* [2016] Random selection weighted by rewards (r)

$$proba[o] = p_{min} + (1 - |O| * p_{min}) * \frac{r[o]}{\sum r}$$

- **Pursuit** Goëffon *et al.* [2016] Selection in favor of the best operator (b)

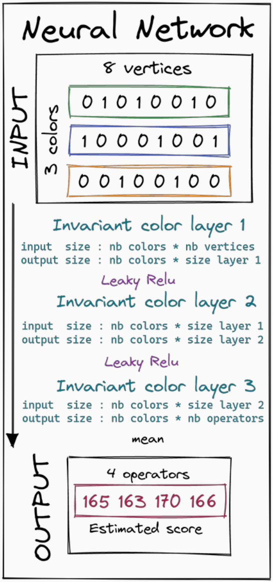
$$proba[b] = proba[b] + \beta(p\_max - proba[b])$$
$$proba[o] = proba[o] + \beta(p\_min - proba[o])$$

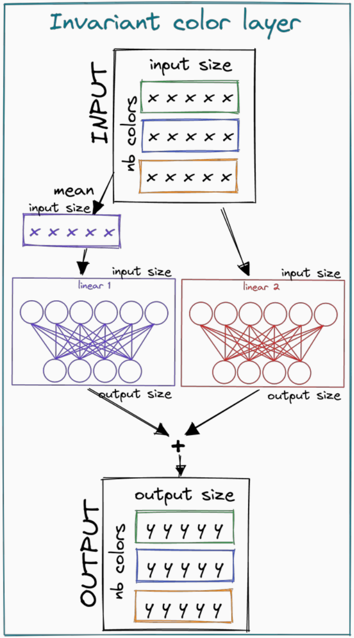
- **UCB** : Focusing on the best while encouraging exploration

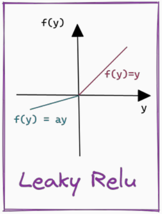
$$score[o] = r[o] + c\sqrt{\frac{2 * \log(\sum visits)}{visits[o]}}$$

- **NN** : Recommendation of a neural network on a raw solution with Deep Sets (Zaheer *et al.* [2017])

Neural Network - NN - Deep sets







Results

WVCP - Comparison between methods

1 point/instance if the average is significantly better for the method on the line compared to the one in the column (Wilcoxon signed-rank, p-value < 0.001)

/48	MCTS+UCB	RedLS	ILS-TS	HEAD+RedLS	HEAD+ILS-TS	Random	Roulette	Deleter	UCB	Pursuit	NN	# BKS	# Best	# Best Avg
MCTS+UCB	-	**25**	**15**	3	**20**	0	0	0	0	0	0	20	19	15
RedLS	11	-	10	9	14	9	9	9	9	9	9	15	24	11
ILS-TS	10	**27**	-	8	**19**	3	6	5	4	1	3	23	25	21
HEAD+RedLS	**16**	**26**	**15**	-	**25**	1	1	0	0	1	0	19	19	11
HEAD+ILS-TS	5	**20**	6	5	-	0	0	0	0	0	0	18	19	13
Random	**19**	**27**	**20**	**10**	**25**	-	0	0	0	0	0	21	22	19
Roulette	**17**	**26**	**20**	**9**	**26**	0	-	0	0	0	0	22	22	17
Deleter	**20**	**26**	**19**	**9**	**26**	**3**	0	-	0	0	0	**24**	**28**	19
UCB	**20**	**26**	**20**	**9**	**26**	**1**	**1**	0	-	0	0	23	23	19
Pursuit	**19**	**26**	**23**	**11**	**26**	**1**	0	0	0	-	0	**24**	26	**22**
NN	**20**	**27**	**21**	**10**	**27**	0	**1**	0	0	0	-	21	23	19

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Deleter	**20**	**26**	**19**	**9**	**26**	**3**	0	-	0	0	0	**24**	**28**	19
UCB	**20**	**26**	**20**	**9**	**26**	**1**	**1**	0	-	0	0	23	23	19
Pursuit	**19**	**26**	**23**	**11**	**26**	**1**	0	0	0	-	0	**24**	26	**22**
NN	**20**	**27**	**21**	**10**	**27**	0	**1**	0	0	0	-	21	23	19

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RedLS	11	-	10	9	14	9	9	9	9	9	9	15	24	11
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HEAD+ILS-TS	5	**20**	6	5	-	0	0	0	0	0	0	18	19	13
Random	**19**	**27**	**20**	**10**	**25**	-	0	0	0	0	0	21	22	19
Roulette	**17**	**26**	**20**	**9**	**26**	0	-	0	0	0	0	22	22	17
Deleter	**20**	**26**	**19**	**9**	**26**	**3**	0	-	0	0	0	**24**	**28**	19
UCB	**20**	**26**	**20**	**9**	**26**	**1**	**1**	0	-	0	0	23	23	19
Pursuit	**19**	**26**	**23**	**11**	**26**	**1**	0	0	0	-	0	**24**	26	**22**
NN	**20**	**27**	**21**	**10**	**27**	0	**1**	0	0	0	-	21	23	19

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MCTS+UCB	-	**25**	**15**	3	**20**	0	0	0	0	0	0	20	19	15
RedLS	11	-	10	9	14	9	9	9	9	9	9	15	24	11
ILS-TS	10	**27**	-	8	**19**	3	6	5	4	1	3	23	25	21
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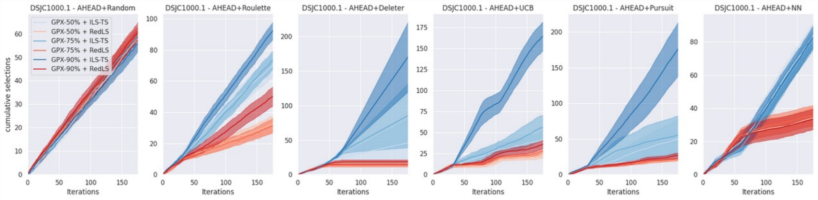
Deleter	**20**	**26**	**19**	**9**	**26**	**3**	0	-	0	0	0	**24**	**28**	19
UCB	**20**	**26**	**20**	**9**	**26**	**1**	**1**	0	-	0	0	23	23	19
Pursuit	**19**	**26**	**23**	**11**	**26**	**1**	0	0	0	-	0	**24**	26	**22**
NN	**20**	**27**	**21**	**10**	**27**	0	**1**	0	0	0	-	21	23	19

WVCP - Results

instance	BKS	RedLS			ILS-TS			HEAD+RedLS			AHEAD+Random			AHEAD+Deleter		
		best	mean	time	best	mean	time	best	mean	time	best	mean	time	best	mean	time
C2000.5	2144	**2131**	**2155.7**	18367	2244	2264.4	6423	2244	2257.9	7453	2220	2236.8	12962	2218	2236.3	1782
C2000.9	5477	**5439**	**5455.1**	23137	5847	5910.1	23014	5732	5748.2	12980	5732	5783.9	12491	5717	5758.8	12327
DSJC1000.1	300	303	306.9	5839	305	306.2	5819	304	305.6	7380	302	303.8	9348	**300**	**302.2**	12874
DSJC1000.5	1185	**1190**	**1206.9**	12204	1241	1267.7	21935	1225	1229.7	7011	1222	1228.2	5371	1224	1230.5	1476
DSJC1000.9	2836	**2828**	**2841.8**	22796	3004	3035.9	25345	2909	2926.5	820	2911	2928.7	12633	2907	2926.8	2379
DSJC500.1	184	187	194	702	185	187.3	7107	186	186.9	6594	185	186.5	10290	**184**	185.9	8022
DSJC500.5	685	707	712.5	27147	711	721.2	9150	709	712.6	2534	**706**	**711.5**	12516	709	713.5	5838
DSJC500.9	1662	**1667**	**1671**	9925	1709	1725.3	24351	1680	1683.5	4053	1678	1684.2	12644	1676	1682.8	8149
DSJC250.1	127	129	131.4	56	**127**	127.1	11901	**127**		4516	**127**		3729	**127**	127.2	3235
DSJC250.5	392	399	400.8	2602	**392**	**393.9**	10722	395	396.2	8349	393	395.2	9592	**392**	396.6	6028
DSJC250.9	934*	**934**	935	9679	**934**	935.1	14740	**934**	935.1	6741	**934**	**934.2**	8097	**934**	935	5011
flat1000_50_0	924	**1152**	**1165.7**	6259	1213	1230.5	570	1181	1187.7	7544	1179	1186.3	4428	1180	1186.8	2952
flat1000_60_0	1162	**1196**	**1204.8**	1877	1247	1263.8	25765	1216	1227.2	10824	1213	1223.7	11726	1217	1224.5	9840
flat1000_76_0	1165	**1163**	**1183.2**	28084	1228	1242.2	16513	1192	1204	2214	1187	1203	10742	1196	1204	8938
latin_square_10	1480	**1505**	**1515.3**	14189	1555	1575	18924	1523	1532.5	11286	1510	1526.2	13987	1517	1527.8	8732
le450_15a	212	213	215.4	54	**211**	213.6	11684	**212**	212.8	6777	**212**	212.8	8819	**211**	**212.4**	10557
le450_15b	216	218	219.9	41	217	217.1	10346	**216**	217	3204	**216**	217.1	2736	**215**	**216.5**	11124
le450_15c	275	282	285.4	82	279	281.7	16288	277	279.4	8360	277	**278.8**	7220	278	279.4	4788
le450_15d	272	277	280.6	325	275	277.6	8456	274	276.1	6004	274	275.6	8759	**273**	275.2	13299
le450_25a	306	**306**	306.6	2881	**306**		142	**306**		161	**306**		169	**306**		131
le450_25b	307*	**307**	307.6	95	**307**		23	**307**		53	**307**		28	**307**		19
le450_25c	342	348	352.8	583	348	349.1	16413	347	348.1	180	**346**	**347.8**	5652	**346**	348	588
le450_25d	330	335	339.4	232	337	338.7	14212	333	334.4	5904	333	334.2	6282	333	334.2	9648
queen14_14	215	218	223.8	568	**215**	216.4	9862	216	216.6	7956	**215**	216.2	6384	**214**	**215.3**	8624
wap01a	545	557	577	995	**547**	**550.1**	20531	552	559.1	8178	549	553.6	14094	549	552.8	8874
wap02a	538	554	572.1	16183	**536**	**541**	21912	550	557.1	13884	541	546.1	7654	541	545.5	12994
wap03a	562	**569**	575.5	17878	572	575.5	22637	577	579.7	6992	573	576.3	8096	573	575.9	2944
wap04a	563	**567**	578.9	13939	**567**	**570.5**	7346	573	575.6	3152	570	573.2	1970	569	572.5	13790
wap05a	541	**542**	543.8	7719	**542**	**542.2**	11809	**542**	542.9	4471	**542**	543	12056	**542**	543.2	2772
wap06a	516	519	526.1	1575	**516**	**519.5**	6264	519	520.7	12180	518	521	9100	520	521.2	5978
wap07a	555	**554**	573	8460	565	569.2	16299	557	559.4	3360	558	559.8	12040	557	**559.2**	12460
wap08a	529	**536**	543.7	19557	543	546.9	19271	539	540.8	7452	539	541.2	1800	538	**540.1**	10608
#BKS			15/48			23/48			19/48			21/48			24/48	
#Best			24/48			25/48			19/48			22/48			28/48	
#Best Avg			11/48			21/48			11/48			19/48			19/48	

WVCP - Selections

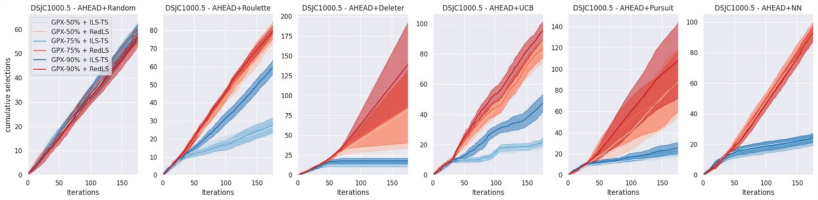


No large differences in selections between the different crossovers.

ILS-TS

RedLS

Choice of the local search is more important.



GCP - Comparison between methods

1 point/instance if the average is significantly better for the method on the line compared to the one in the column (Wilcoxon signed-rank, p-value < 0.001)

/31	PartialCol	TabuCol	HEAD+PC	HEAD+TC	Random	Roulette	Deleter	UCB	Pursuit	NN	# BKS	# Best	# Best Avg
PartialCol	-	2	3	2	1	2	1	2	2	2	5	8	11
TabuCol	**14**	-	**11**	2	2	1	0	2	0	1	8	14	7
HEAD+PC	**8**	6	-	1	0	0	1	0	0	0	6	10	7
HEAD+TC	**18**	**12**	**20**	-	**4**	**2**	1	**2**	2	2	7	17	15
Random	**17**	**11**	**19**	1	-	0	1	1	0	0	9	17	9
Roulette	**17**	**11**	**19**	1	0	-	0	0	0	0	11	19	12
Deleter	**19**	**15**	**20**	**5**	**8**	**3**	-	**5**	**1**	**1**	**13**	**24**	**20**
UCB	**19**	**11**	**20**	1	1	0	0	-	0	0	10	18	10
Pursuit	**19**	**13**	**20**	**3**	**5**	**2**	0	**1**	-	0	11	20	14
NN	**19**	**12**	**20**	2	**4**	0	0	0	0	-	12	23	16

PC = PartialCol – TC = TabuCol

GCP - Comparison between methods

1 point/instance if the average is significantly better for the method on the line compared to the one in the column (Wilcoxon signed-rank, p-value < 0.001)

/31	PartialCol	TabuCol	HEAD+PC	HEAD+TC	Random	Roulette	Deleter	UCB	Pursuit	NN	# BKS	# Best	# Best Avg
PartialCol	-	2	3	2	1	2	1	2	2	2	5	8	11
TabuCol	**14**	-	**11**	2	2	1	0	2	0	1	8	14	7
HEAD+PC	**8**	6	-	1	0	0	1	0	0	0	6	10	7
HEAD+TC	**18**	**12**	**20**	-	**4**	**2**	1	**2**	2	2	7	17	15
Random	**17**	**11**	**19**	1	-	0	1	1	0	0	9	17	9
Roulette	**17**	**11**	**19**	1	0	-	0	0	0	0	11	19	12
Deleter	**19**	**15**	**20**	**5**	**8**	**3**	-	**5**	**1**	**1**	**13**	**24**	**20**
UCB	**19**	**11**	**20**	1	1	0	0	-	0	0	10	18	10
Pursuit	**19**	**13**	**20**	**3**	**5**	**2**	0	**1**	-	0	11	20	14
NN	**19**	**12**	**20**	2	**4**	0	0	0	0	-	12	23	16

PC = PartialCol – TC = TabuCol

GCP - Comparison between methods

1 point/instance if the average is significantly better for the method on the line compared to the one in the column (Wilcoxon signed-rank, p-value < 0.001)

/31	PartialCol	TabuCol	HEAD+PC	HEAD+TC	Random	Roulette	Deleter	UCB	Pursuit	NN	♯ BKS	♯ Best	♯ Best Avg
PartialCol	-	2	3	2	1	2	1	2	2	2	5	8	11
TabuCol	**14**	-	**11**	2	2	1	0	2	0	1	8	14	7
HEAD+PC	**8**	6	-	1	0	0	1	0	0	0	6	10	7
HEAD+TC	**18**	**12**	**20**	-	**4**	**2**	1	**2**	2	2	7	17	15
Random	**17**	**11**	**19**	1	-	0	1	1	0	0	9	17	9
Roulette	**17**	**11**	**19**	1	0	-	0	0	0	0	11	19	12
Deleter	**19**	**15**	**20**	**5**	**8**	**3**	-	**5**	**1**	**1**	**13**	**24**	**20**
UCB	**19**	**11**	**20**	1	1	0	0	-	0	0	10	18	10
Pursuit	**19**	**13**	**20**	**3**	**5**	**2**	0	**1**	-	0	11	20	14
NN	**19**	**12**	**20**	2	**4**	0	0	0	0	-	12	23	16

PC = PartialCol – TC = TabuCol

GCP - Comparison between methods

1 point/instance if the average is significantly better for the method on the line compared to the one in the column (Wilcoxon signed-rank, p-value < 0.001)

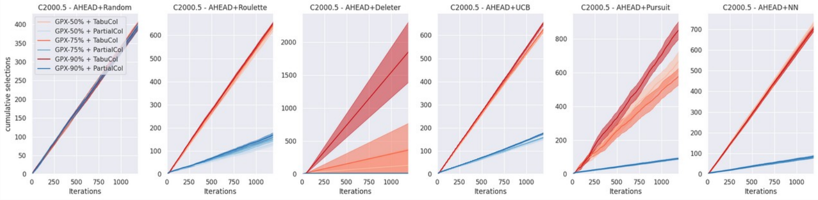
/31	PartialCol	TabuCol	HEAD+PC	HEAD+TC	Random	Roulette	Deleter	UCB	Pursuit	NN	# BKS	# Best	# Best Avg
PartialCol	-	2	3	2	1	2	1	2	2	2	5	8	11
TabuCol	**14**	-	**11**	2	2	1	0	2	0	1	8	14	7
HEAD+PC	**8**	6	-	1	0	0	1	0	0	0	6	10	7
HEAD+TC	**18**	**12**	**20**	-	**4**	**2**	1	**2**	2	2	7	17	15
Random	**17**	**11**	**19**	1	-	0	1	1	0	0	9	17	9
Roulette	**17**	**11**	**19**	1	0	-	0	0	0	0	11	19	12
Deleter	**19**	**15**	**20**	**5**	**8**	**3**	-	**5**	**1**	**1**	**13**	**24**	**20**
UCB	**19**	**11**	**20**	1	1	0	0	-	0	0	10	18	10
Pursuit	**19**	**13**	**20**	**3**	**5**	**2**	0	**1**	-	0	11	20	14
NN	**19**	**12**	**20**	2	**4**	0	0	0	0	-	12	23	16

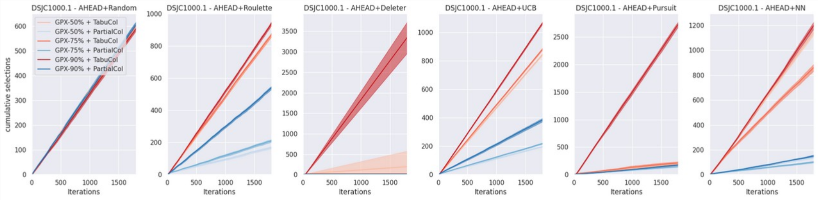
PC = PartialCol – TC = TabuCol

GCP - Results

instance	BKS	PartialCol			TabuCol			HEAD+TC			AHEAD+Random			AHEAD+Deleter		
		best	mean	time	best	mean	time	best	mean	time	best	mean	time	best	mean	time
C2000.5	145	164	165.2	5313	162	162.8	4628	**148**	**149.2**	3330	150	150.7	3101	149	150.7	3152
C2000.9	408	420	420.8	5171	411	412.5	4786	405	406.4	2328	405	407.7	2956	**404**	**405.6**	2988
C4000.5	259	304	305.6	6690	303	304.2	5567	**278**	**279.6**	3580	280	281.6	3651	279	280.8	3404
DSJC500.1	12	**12**		128	**12**		75	**12**		86	**12**		80	**12**		56
DSJC500.5	47	50	50.1	2227	49		460	**48**		819	**48**		1258	**48**		850
DSJC500.9	126	128		975	**126**	126.3	2988	**126**		1027	**126**	126.1	1379	**126**		632
DSJC1000.1	20	21		1	21		0	21		0	21		1	**20**	**20.9**	2391
DSJC1000.5	82	90	90.5	3516	88		1760	**83**	**83.3**	2290	**83**	83.5	2372	**83**	83.5	2511
DSJC1000.9	222	227	228.4	3630	224	224.9	3345	**223**	224	1616	**223**	224.2	2734	**223**	**223.8**	1589
DSJR500.5	122*	125	126.2	1666	124	127	1155	**123**	124	1766	**123**	124.2	2245	**123**	**123.8**	2289
flat300_28_0	28*	**28**		896	**28**	29.5	3220	30	30.8	1916	**28**	28.5	702	**28**	30.4	5
flat1000_50_0	50*	**50**		44	**50**		69	**50**		28	**50**		8	**50**		8
flat1000_60_0	60*	**60**		213	**60**		233	**60**		54	**60**		28	**60**		29
flat1000_76_0	76*	89	89.1	2845	86	87	3096	**82**	**82.3**	1905	**82**	82.8	2775	**82**	82.8	1969
latin_square_10	97	107	110.2	4875	100	100.8	4377	102	103.7	93	103	103.8	1996	**99**	**100.7**	1729
le450_25c	25*	27		69	26		0	26		0	**25**	25.9	1407	**25**	**25.3**	1022
le450_25d	25*	27		50	26		0	26		0	26		0	**25**	**25.3**	1537
r250.5	65*	67		134	66	67.2	462	**65**	66	3378	**65**	66	1638	66		549
r1000.1c	98	141	149.1	61	134	155.2	77	**100**	101.6	264	**100**	101.6	1674	**100**	101.6	1621
r1000.5	234	247	248.1	5638	**244**	245.6	3622	246	247.6	1479	246	247.4	2134	245	**245.5**	2009
wap01a	41*	42		1088	42	43	2160	42		137	42		143	**41**	**42**	1958
wap02a	40*	41	41.7	4275	**40**	41.1	6499	41		15	41		15	**40**	**40.8**	1634
wap03a	43	44		91	44	45.9	4342	45		261	45		87	**43**	44.3	2387
wap04a	41	43		61	**42**	43.1	4869	43		880	43		1186	43		293
wap06a	40*	41		98	**40**	41.3	4248	**40**		909	**40**	40.8	1549	**40**		246
wap07a	41	44		41	**41**	42.3	5046	42	42.1	1771	42	43	2526	42	42.1	494
wap08a	40*	43	43.2	2750	**41**	**41.5**	2967	42		48	42		365	**41**	41.9	2146
#BKS		5/31			8/31			7/31			9/31			13/31		
#Best		8/31			14/31			17/31			17/31			24/31		
#Best Avg		11/31			7/31			15/31			9/31			20/31		

GCP - Selections





PartialCol

TabuCol

Conclusion

AHEAD - Conclusion

GCP - Conclusion

- HEAD + LS improves LS results
- AHEAD better than HEAD but HEAD+TabuCol often very good
- New best score on C2000.9 (Success : 404 Found)

WVCP - Conclusion

- HEAD + LS improves RedLS but not ILS-TS
- RedLS stay better on 9 instances and ILS-TS on less than 6 (/48)
- New scores : le450_15a (211), le450_15b (215), queen14_14 (214)

To summarize

- No large differences in selections
- Local Search choice more important
- Deleter and Pursuit get the best results

Thank you for your attention !

Questions ?

Source code, results tables, articles :

https://cyril-grelier.github.io/

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