

# A memetic algorithm with adaptive operator selection for graph coloring

AHEAD (Adaptive HEAD) Algorithm

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Cyril Grelier - Olivier Goudet - Jin-Kao Hao  
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Université d'Angers – LERIA



## Studied Problems

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# GCP - Graph Coloring Problem

## Graph Coloring

Objective: find a legal color that minimizes the number of colors

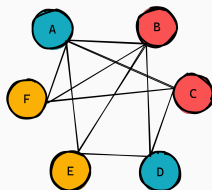
Score :

- Number of colors  $k$  (legal)
- Number of conflicts  $|C|$  (illegal)
- Number of uncolored vertices  $|U|$  (partially legal)

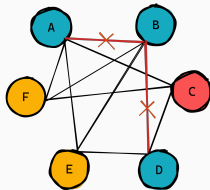
NP-Hard problem

Applications :

- Scheduling problems
- Register allocation
- Sudoku
- ...



✓ legal coloring  
with 3 colors



✗ Unsatisfied constraints  
2 conflicts

# WVCP - Weighted Vertex Coloring Problem

## Weighted Vertex Coloring

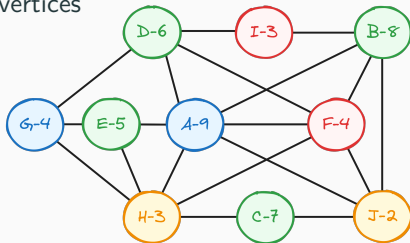
Objective: find a legal coloring that minimizes the sum of the weights of the heaviest vertices in each color

$$\text{Score} : \sum_{i=1}^k \max_{v \in V_i} w(v)$$

NP-Hard problem

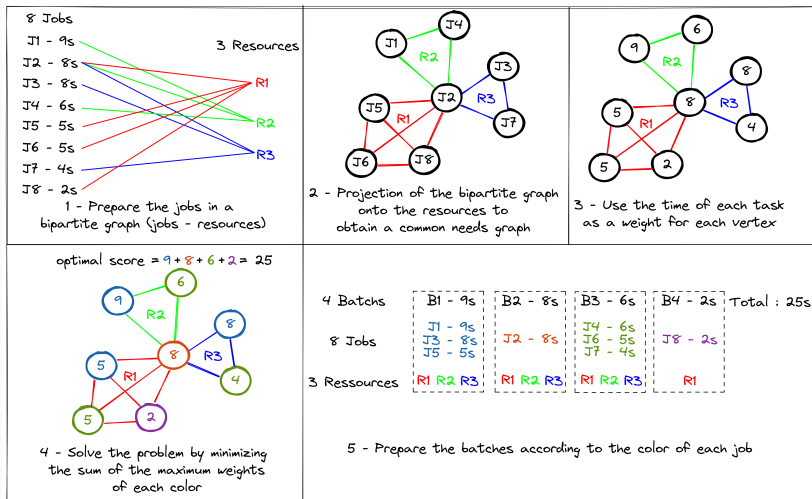
Applications :

- Traffic management in satellite communications
- Matrix decomposition problem
- Scheduling batch job in parallel



$$\text{Score} = \begin{array}{|c|} \hline 9 \\ \hline 4 \\ \hline \end{array} + \begin{array}{|c|} \hline 8 \\ \hline 7 \\ \hline 6 \\ \hline 5 \\ \hline \end{array} + \begin{array}{|c|} \hline 4 \\ \hline 3 \\ \hline \end{array} + \begin{array}{|c|} \hline 3 \\ \hline 2 \\ \hline \end{array} = 24$$

# WVCP - Scheduling Batch Job in Parallel



## State of the Art

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- Local Search:
  - **TabuCol** Hertz et Werra [1987] : illegal, one-move
  - **PartialCol** Blöchliger et Zufferey [2008] : partial legal, grenade
  - **ILS** Chiarandini et Stützle [2002] : perturbations, acceptance criteria
- Memetic Algorithms :
  - **HEA** Galinier et Hao [1999] : GPX, TabuCol
  - **Evo-Div** Porumbel *et al.* [2010] : multi-parents crossover, distances
  - **MACOL** Lü et Hao [2010] : multi-parents crossover, distances
  - **HEAD** Moalic et Gondran [2018] : 2 individuals, GPX, TabuCol
  - **DLMCOL** Goudet *et al.* [2022] : +20 000, NN select crossover
  - **AHEAD** Grelier *et al.* [2024] : Adaptive HEAD
- Learning :
  - **PLSCOL** Zhou *et al.* [2018] : local search, reinforcement learning
  - **TensCol** Goudet *et al.* [2021] : tensor, gradient descent
  - **NRPA** Cazenave *et al.* [2021] : MCTS, sequence, gradient descent

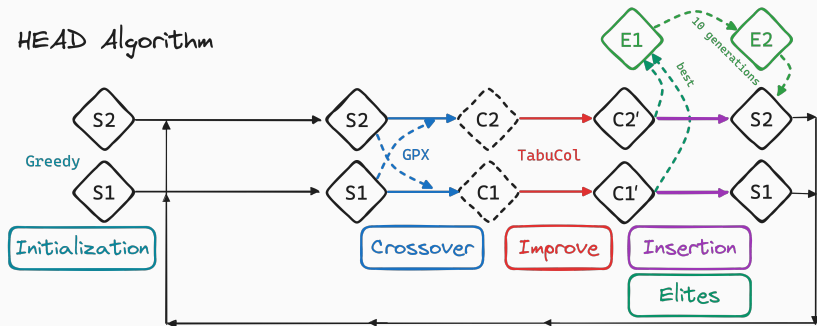
- Learning :
  - **MCTS + Local Search** Grelier *et al.* [2022] LS as simulation
  - **MCTS + Hyperheuristics** Grelier *et al.* [2023] : select LS
- Memetic Algorithms :
  - **DLMLCOL** Goudet *et al.* [2022] : +20000, NN select crossover
  - **AHEAD** Grelier *et al.* [2024] : Adaptive HEAD
- Local Search :
  - **AFISA** Sun *et al.* [2018] : illegal, one-move, adaptive coefficient
  - **RedLS** Wang *et al.* [2020] : illegal, weighted edges, perturbations
  - **ILS-TS** Nogueira *et al.* [2021] : p-legal, 6 neighbors, perturbations
  - **TabuWeight** Grelier *et al.* [2022] : legal, one-move
- Exact Methods :
  - **2-Phase** Malaguti *et al.* [2009] : column generation, ILP
  - **MWSS** Cornaz *et al.* [2017] : MIP, max weight stable set problem
  - **CP** Goudet *et al.* [2023] : 3 CP models, reduction



# Memetic Algorithm – AHEAD

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# HEAD - Hybrid Evolutionary Algorithm in Duet



Moalic et Gondran [2018] – Variations on memetic algorithms for graph coloring problems

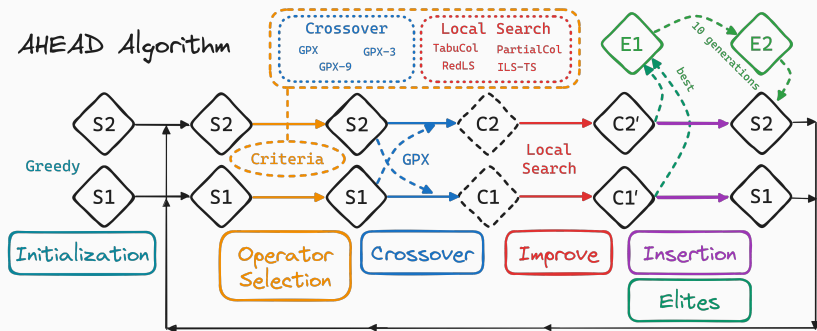
## Why HEAD?

- One of the best algorithm for GCP
- Simple and efficient

## How?

- 2 individuals
- GPX
- TabuCol

# AHEAD - Adaptive Hybrid Evolutionary Algorithm in Duet



## Why AHEAD?

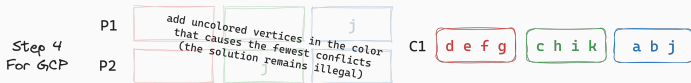
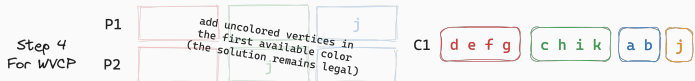
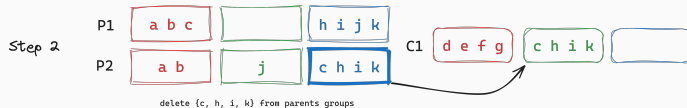
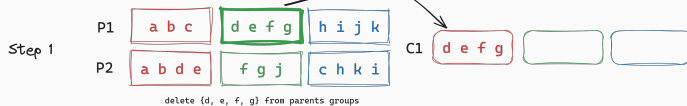
- Attempt to improve HEAD
- Adapt to the instance

## How?

- Hyperheuristics
- Multiple crossover
- Multiple local search

# GPX - Galinier et Hao [1999]

## GPX crossover - Greedy Partition Crossover



## Crossover

- **GPX** : 1 color in P1 for 1 color in P2
- **GPX-3** : 3 colors in P1 for 1 color in P2
- **GPX-9** : 9 colors in P1 for 1 color in P2

## Local Search - GCP

- **TabuCol** : Hertz et Werra [1987]
- **PartialCol** : Blöchliger et Zufferey [2008]

## Local Search - WVCP

- **RedLS** : Wang *et al.* [2020]
- **ILS-TS** : Nogueira *et al.* [2021]

## Why?

- Diversification during the search
- No best operator on all instances
- Adapt to the instance

## How?

- Selection criteria : Learn to select the best operator
- Reward : Score of the solution after the local search
- Selection : pair <crossover, local search>
- Exception : NN : Generate all crossovers and select the best one

## Selection criteria

- **Random** Uniform random choice
- **Deleter** Delete the least performing operators ( $o$ )
- **Roulette** Goëffon *et al.* [2016] Random selection weighted by rewards ( $r$ )

$$proba[o] = p_{min} + (1 - |O| * p_{min}) * \frac{r[o]}{\sum r}$$

- **Pursuit** Goëffon *et al.* [2016] Selection in favor of the best operator ( $b$ )

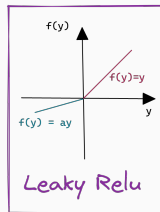
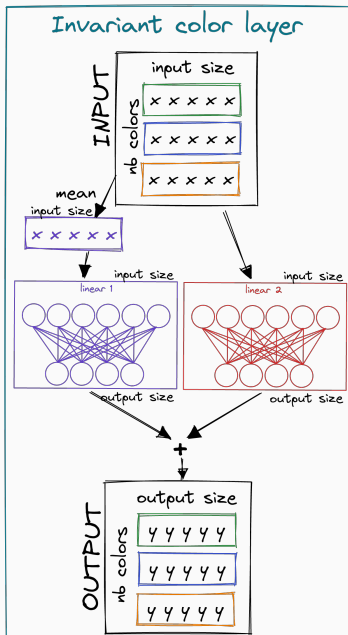
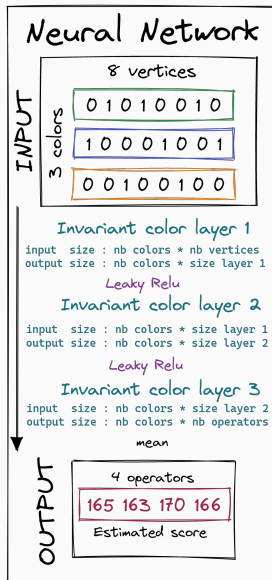
$$\begin{cases} proba[b] = proba[b] + \beta(p_{max} - proba[b]) \\ proba[o] = proba[o] + \beta(p_{min} - proba[o]) \end{cases}$$

- **UCB** : Focusing on the best while encouraging exploration

$$score[o] = r[o] + c * \sqrt{2 * \frac{\log(\sum visits)}{visits[o]}}$$

- **NN** : Recommendation of a neural network on a raw solution with Deep Sets (Zaheer *et al.* [2017])

# Neural Network - NN - Deep sets





# AHEAD - Results WVCP

1 point/instance if the average is significantly better for the method on the line compared to the one in the column (Wilcoxon signed-rank, p-value < 0.001)

/48	MCTS+UCB	RedLS	ILS-TS	HEAD+RedLS	HEAD+ILS-TS	Random	Roulette	Deleter	UCB	Pursuit	NN	# BKS	# Best	# Best Avg
MCTS+UCB	-	<b>25</b>	<b>15</b>	3	<b>20</b>	0	0	0	0	0	0	20	19	15
RedLS	11	-	10	9	14	9	9	9	9	9	9	15	24	11
ILS-TS	10	<b>27</b>	-	8	<b>19</b>	3	6	5	4	1	3	23	25	21
HEAD+RedLS	<b>16</b>	<b>26</b>	<b>15</b>	-	<b>25</b>	1	1	0	0	1	0	19	19	11
HEAD+ILS-TS	5	<b>20</b>	6	5	-	0	0	0	0	0	0	18	19	13
Random	<b>19</b>	<b>27</b>	<b>20</b>	<b>10</b>	<b>25</b>	-	0	0	0	0	0	21	22	19
Roulette	<b>17</b>	<b>26</b>	<b>20</b>	<b>9</b>	<b>26</b>	0	-	0	0	0	0	22	22	17
Deleter	<b>20</b>	<b>26</b>	<b>19</b>	<b>9</b>	<b>26</b>	<b>3</b>	0	-	0	0	0	<b>24</b>	<b>28</b>	19
UCB	<b>20</b>	<b>26</b>	<b>20</b>	<b>9</b>	<b>26</b>	<b>1</b>	<b>1</b>	0	-	0	0	23	23	19
Pursuit	<b>19</b>	<b>26</b>	<b>23</b>	<b>11</b>	<b>26</b>	<b>1</b>	0	0	0	-	0	<b>24</b>	26	<b>22</b>
NN	<b>20</b>	<b>27</b>	<b>21</b>	<b>10</b>	<b>27</b>	0	<b>1</b>	0	0	0	-	21	23	19

# AHEAD - Results **GCP**

1 point/instance if the average is significantly better for the method on the line compared to the one in the column (Wilcoxon signed-rank, p-value < 0.001)

/31	PartialCol	TabuCol	HEAD+PC	HEAD+TC	Random	Roulette	Deleter	UCB	Pursuit	NN	# BKS	# Best	# Best Avg
PartialCol	-	2	3	2	1	2	1	2	2	2	5	8	11
TabuCol	<b>14</b>	-	<b>11</b>	2	2	1	0	2	0	1	8	14	7
HEAD+PC	<b>8</b>	6	-	1	0	0	1	0	0	0	6	10	7
HEAD+TC	<b>18</b>	<b>12</b>	<b>20</b>	-	<b>4</b>	<b>2</b>	<b>1</b>	<b>2</b>	<b>2</b>	<b>2</b>	7	17	15
Random	<b>17</b>	<b>11</b>	<b>19</b>	1	-	0	1	1	0	0	9	17	9
Roulette	<b>17</b>	<b>11</b>	<b>19</b>	1	0	-	0	0	0	0	11	19	12
Deleter	<b>19</b>	<b>15</b>	<b>20</b>	<b>5</b>	<b>8</b>	<b>3</b>	-	<b>5</b>	<b>1</b>	<b>1</b>	<b>13</b>	<b>24</b>	<b>20</b>
UCB	<b>19</b>	<b>11</b>	<b>20</b>	1	1	0	0	-	0	0	10	18	10
Pursuit	<b>19</b>	<b>13</b>	<b>20</b>	<b>3</b>	<b>5</b>	<b>2</b>	0	<b>1</b>	-	0	11	20	14
NN	<b>19</b>	<b>12</b>	<b>20</b>	2	<b>4</b>	0	0	0	0	-	12	23	16

## About the selection criteria

- No changes in the selections
- More conservative GPX Crossover often preferred but without very large differences
- Local Search more important

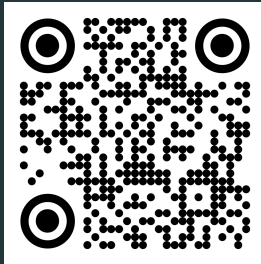
## To summarize

- Local Search alone keeps some instances with better results
- Better results with AHEAD than other methods in general
- Deleter often better with new best scores :
  - **GCP** : C2000.9 (404)
  - **WVCP** : le450\_15a (211), le450\_15b (215), queen14\_14 (214)

# Thank you for your attention !

## Questions ?

Source code, results tables, articles :



<https://cyril-grelier.github.io/>

## References

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**AHEAD**

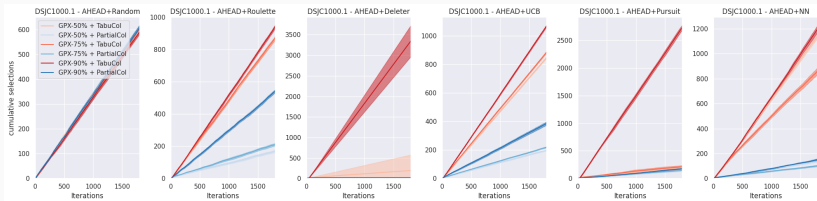
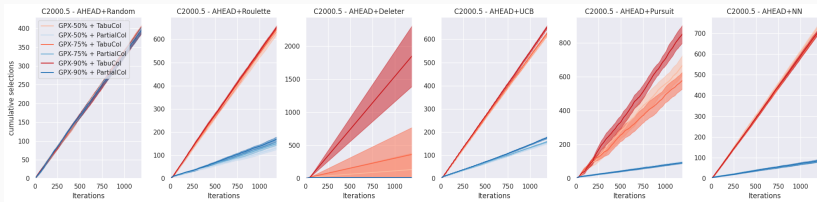
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# AHEAD - Results **GCP**

instance	BKS	PartialCol			TabuCol			HEAD+TC			AHEAD+Random			AHEAD+Deleter		
		best	mean	time	best	mean	time	best	mean	time	best	mean	time	best	mean	time
C2000.5	145	164	165.2	5313	162	162.8	4628	<b>148</b>	<b>149.2</b>	3330	150	150.7	3101	149	150.7	3152
C2000.9	408	420	420.8	5171	411	412.5	4786	<b>405</b>	406.4	2328	<b>405</b>	407.7	2956	<b>404</b>	<b>405.6</b>	2988
C4000.5	259	304	305.6	6690	303	304.2	5567	<b>278</b>	<b>279.6</b>	3580	280	281.6	3651	279	280.8	3404
DSJC500.1	12	<b>12</b>		128	<b>12</b>		75	<b>12</b>		86	<b>12</b>		80	<b>12</b>		56
DSJC500.5	47	50	50.1	2227	49		460	<b>48</b>		819	<b>48</b>		1258	<b>48</b>		850
DSJC500.9	126	128		975	<b>126</b>	126.3	2988	<b>126</b>		1027	<b>126</b>	126.1	1379	<b>126</b>		632
DSJC1000.1	20	21		1	21		0	21		0	21		1	<b>20</b>	<b>20.9</b>	2391
DSJC1000.5	82	90	90.5	3516	88		1760	<b>83</b>	<b>83.3</b>	2290	<b>83</b>	83.5	2372	<b>83</b>	83.5	2511
DSJC1000.9	222	227	228.4	3630	224	224.9	3345	<b>223</b>	224	1616	<b>223</b>	224.2	2734	<b>223</b>	<b>223.8</b>	1589
DSJR500.5	122*	125	126.2	1666	124	127	1155	<b>123</b>	124	1766	<b>123</b>	124.2	2245	<b>123</b>	<b>123.8</b>	2289
flat300_28_0	28*	<b>28</b>		896	<b>28</b>	29.5	3220	30	30.8	1916	<b>28</b>	28.5	702	<b>28</b>	30.4	5
flat1000_50_0	50*	<b>50</b>		44	<b>50</b>		69	<b>50</b>		28	<b>50</b>		8	<b>50</b>		8
flat1000_60_0	60*	<b>60</b>		213	<b>60</b>		233	<b>60</b>		54	<b>60</b>		28	<b>60</b>		29
flat1000_76_0	76*	89	89.1	2845	86	87	3096	<b>82</b>	<b>82.3</b>	1905	<b>82</b>	82.8	2775	<b>82</b>	82.8	1969
latin_square_10	97	107	110.2	4875	100	100.8	4377	102	103.7	93	103	103.8	1996	<b>99</b>	<b>100.7</b>	1729
le450_25c	25*	27		69	26		0	26		0	<b>25</b>	25.9	1407	<b>25</b>	<b>25.3</b>	1022
le450_25d	25*	27		50	26		0	26		0	<b>25</b>		0	<b>25</b>	<b>25.3</b>	1537
r250.5	65*	67		134	66	67.2	462	<b>65</b>	66	3378	<b>65</b>	66	1638	66		549
r1000.1c	98	141	149.1	61	134	155.2	77	<b>100</b>	101.6	264	<b>100</b>	101.6	1674	<b>100</b>	101.6	1621
r1000.5	234	247	248.1	5638	<b>244</b>	245.6	3622	246	247.6	1479	246	247.4	2134	245	<b>245.5</b>	2009
wap01a	41*	42		1088	42	43	2160	42		137	42		143	<b>41</b>	<b>42</b>	1958
wap02a	40*	41	41.7	4275	<b>40</b>	41.1	6499	41		15	41		15	<b>40</b>	<b>40.8</b>	1634
wap03a	43	44		91	44	45.9	4342	45		261	45		87	<b>43</b>	44.3	2387
wap04a	41	43		61	<b>42</b>	43.1	4869	43		880	43		1186	43		293
wap06a	40*	41		98	<b>40</b>	41.3	4248	<b>40</b>		909	<b>40</b>	40.8	1549	<b>40</b>		246
wap07a	41	44		41	<b>41</b>	42.3	5046	42	42.1	1771	42	43	2526	42	42.1	494
wap08a	40*	43	43.2	2750	<b>41</b>	<b>41.5</b>	2967	42		48	42		365	<b>41</b>	41.9	2146
#BKS		5/31			8/31			7/31			9/31			13/31		
#Best		8/31			14/31			17/31			17/31			24/31		
#Best Avg		11/31			7/31			15/31			9/31			20/31		



# AHEAD GCP - Selections



# AHEAD - Results WVCP

instance	BKS	RedLS			ILS-TS			HEAD+RedLS			AHEAD+Random			AHEAD+Deleter		
		best	mean	time	best	mean	time	best	mean	time	best	mean	time	best	mean	time
C2000.5	2144	<b>2131</b>	<b>2155.7</b>	18367	2244	2264.4	6423	2244	2257.9	7453	2220	2236.8	12962	2218	2236.3	1782
C2000.9	5477	<b>5439</b>	<b>5455.1</b>	23137	5847	5910.1	23014	5732	5748.2	12980	5732	5783.9	12491	5717	5758.8	12327
DSJC1000.1	300	303	306.9	5839	305	306.2	5819	304	305.6	7380	302	303.8	9348	<b>300</b>	<b>302.2</b>	12874
DSJC1000.5	1185	<b>1190</b>	<b>1206.9</b>	12204	1241	1267.7	21935	1225	1229.7	7011	1222	1228.2	5371	1224	1230.5	1476
DSJC1000.9	2836	<b>2828</b>	<b>2841.8</b>	22796	3004	3035.9	25345	2909	2926.5	820	2911	2928.7	12633	2907	2926.8	2379
DSJC500.1	184	187	194	702	185	187.3	7107	186	186.9	6594	185	186.5	10290	<b>184</b>	185.9	8022
DSJC500.5	685	707	712.5	27147	711	721.2	9150	709	712.6	2534	<b>706</b>	<b>711.5</b>	12516	709	713.5	5838
DSJC500.9	1662	<b>1667</b>	<b>1671</b>	9925	1709	1725.3	24351	1680	1683.5	4053	1678	1684.2	12644	1676	1682.8	8149
DSJC250.1	127	129	131.4	56	<b>127</b>	127.1	11901	<b>127</b>		4516	<b>127</b>		3729	<b>127</b>	127.2	3235
DSJC250.5	392	399	400.8	2602	<b>392</b>	<b>393.9</b>	10722	395	396.2	8349	393	395.2	9592	<b>392</b>	396.6	6028
DSJC250.9	934*	<b>934</b>	935	9679	<b>934</b>	935.1	14740	<b>934</b>	935.1	6741	<b>934</b>	<b>934.2</b>	8097	<b>934</b>	935	5011
flat1000_50_0	924	<b>1152</b>	<b>1165.7</b>	6259	1213	1230.5	570	1181	1187.7	7544	1179	1186.3	4428	1180	1186.8	2952
flat1000_60_0	1162	<b>1196</b>	<b>1204.8</b>	1877	1247	1263.8	25765	1216	1227.2	10824	1213	1223.7	11726	1217	1224.5	9840
flat1000_76_0	1165	<b>1163</b>	<b>1183.2</b>	28084	1228	1242.2	16513	1192	1204	2214	1187	1203	10742	1196	1204	8938
latin_square_10	1480	<b>1505</b>	<b>1515.3</b>	14189	1555	1575	18924	1523	1532.5	11286	1510	1526.2	13987	1517	1527.8	8732
le450_15a	212	213	215.4	54	<b>211</b>	213.6	11684	<b>212</b>	212.8	6777	<b>212</b>	212.8	8819	<b>211</b>	<b>212.4</b>	10557
le450_15b	216	218	219.9	41	217	217.1	10346	<b>216</b>	217	3204	<b>216</b>	217.1	2736	<b>215</b>	<b>216.5</b>	11124
le450_15c	275	282	285.4	82	279	281.7	16288	277	279.4	8360	277	<b>278.8</b>	7220	278	279.4	4788
le450_15d	272	277	280.6	325	275	277.6	8456	274	276.1	6004	274	275.6	8759	<b>273</b>	275.2	13299
le450_25a	306	<b>306</b>	306.6	2881	<b>306</b>		142	<b>306</b>		161	<b>306</b>		169	<b>306</b>		131
le450_25b	307*	<b>307</b>	307.6	95	<b>307</b>		23	<b>307</b>		53	<b>307</b>		28	<b>307</b>		19
le450_25c	342	348	352.8	583	348	349.1	16413	347	348.1	180	<b>346</b>	<b>347.8</b>	5652	<b>346</b>	348	588
le450_25d	330	335	339.4	232	337	338.7	14212	333	334.4	5904	333	334.2	6282	333	334.2	9648
queen14_14	215	218	223.8	568	<b>215</b>	216.4	9862	216	216.6	7956	<b>215</b>	216.2	6384	<b>214</b>	<b>215.3</b>	8624
wap01a	545	557	577	995	<b>547</b>	<b>550.1</b>	20531	552	559.1	8178	549	553.6	14094	549	552.8	8874
wap02a	538	554	572.1	16183	<b>536</b>	<b>541</b>	21912	550	557.1	13884	541	546.1	7654	541	545.5	12994
wap03a	562	<b>569</b>	575.5	17878	572	575.5	22637	577	579.7	6992	573	576.3	8096	573	575.9	2944
wap04a	563	<b>567</b>	578.9	13939	<b>567</b>	<b>570.5</b>	7346	573	575.6	3152	570	573.2	1970	569	572.5	13790
wap05a	541	<b>542</b>	543.8	7719	<b>542</b>	<b>542.2</b>	11809	<b>542</b>	542.9	4471	<b>542</b>	543	12056	<b>542</b>	543.2	2772
wap06a	516	519	526.1	1575	<b>516</b>	<b>519.5</b>	6264	519	520.7	12180	518	521	9100	520	521.2	5978
wap07a	555	<b>554</b>	573	8460	565	569.2	16299	557	559.4	3360	558	559.8	12040	557	<b>559.2</b>	12460
wap08a	529	<b>536</b>	543.7	19557	543	546.9	19271	539	540.8	7452	539	541.2	1800	538	<b>540.1</b>	10608
#BKS			15/48			23/48			19/48			21/48			24/48	
#Best			24/48			25/48			19/48			22/48			28/48	
#Best Avg			11/48			21/48			11/48			19/48			19/48	

# AHEAD WVCP - Selections

