TabuEdges

Guided Local Search for the Graph Coloring Problem

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Studied Problem

GCP - Graph Coloring Problem

Graph Coloring

Objective: find a legal coloring while minimizing the number of colors

Close to the k-coloring problem (fixed number of color)

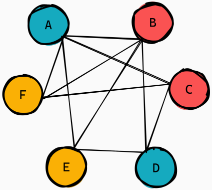
Score :

- Number of colors k (legal)
- Number of conflicts $|C|$ (illegal)
- Number of uncolored vertices $|U|$ (partially legal)

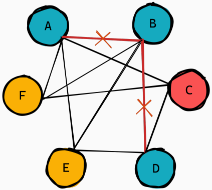
NP-Hard problem

Applications :

- Scheduling problems
- Register allocation
- Sudoku
- ...



legal coloring
with 3 colors



Unsatisfied constraints
2 conflicts

State of the Art

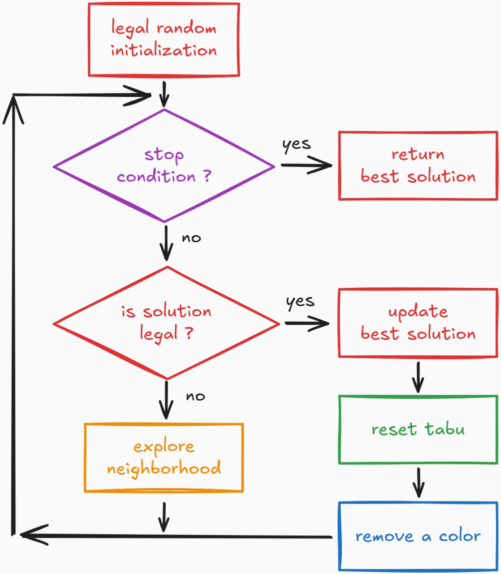
State of the Art - GCP/k-coloring

- Local Search:
  - **TabuCol** Hertz et Werra [1987] : illegal, one-move
  - **PartialCol** Blöchliger et Zufferey [2008] : partial legal, grenade
  - **ILS** Chiarandini et Stützle [2002] : perturbations, acceptance criteria
- Memetic Algorithms :
  - **HEA** Galinier et Hao [1999] : GPX, TabuCol
  - **Evo-Div** Porumbel *et al.* [2010] : multi-parents crossover, distances
  - **MACOL** Lü et Hao [2010] : multi-parents crossover, distances
  - **HEAD** Moalic et Gondran [2018] : 2 individuals, GPX, TabuCol
  - **DLMCOL** Goudet *et al.* [2022] : +20 000, NN select crossover
  - **AHEAD** Grelier *et al.* [2024] : hyperheuristics to select $<$X,LS$>$

- Local Search for the WVCP (Weighted Vertex Coloring Problem) :
  - **RedLS** Wang *et al.* [2020] : illegal, weighted edges, perturbations

TabuEdges

TabuEdges Algorithm



- Color removing
- Tabu Management
- Neighborhood Exploration
- Guided Component

Color Removing

To decrease the number of colors when a legal solution is found

- Fusion: merge two colors groups
- Divide: separate vertices from a color group to multiple color groups

Low impact on the search

Tabu Strategy

Different solutions exists to manage the tabu aspect.

Let's consider the vertex v that move from the color c to another color

$|V|$: number of vertices in the graph

$|C|$: number of conflicts in the solution

- **Tabu List**: size $|V|$ - Hertz et Werra [1987]
  $tabu[v] = turn + random(0, 10) + \alpha * |C|$
- **Tabu Matrix**: size $|V| * k_{max}$ - Moalic et Gondran [2018]
  $tabu[v][c] = turn + random(0, 10) + \alpha * |C|$
- **Configuration Checking**: size $|V|$ - Cai *et al.* [2011]
  $tabu[v] = true$
  $tabu[n] = false, \forall n \in neighbors(v)$

Neighborhood Exploration and Guided Component

Neighborhood:

- 1-opt / one-move : one vertex move to another color

Neighborhood Selection Strategy:

1. TabuCol, Hertz et Werra [1987]
   - select the best non tabu move (can degrade the solution)
   - aspiration critera to accept a tabu move that lead to a better solution
2. TabuEdges
   1. apply a non tabu and non degrading move
   2. if no move is found, penalize conflicting edges*
   3. then move a random vertex in conflict in the least degrading way
3. TabuDouble
   - alternate TabuEdges and TabuCol strategies

***Guided Component**, Wang *et al.* [2020]:

- Increment weight (penality) of the edges in conflict
- Use the weight to compute penalty and delta while searching moves

Experimentations

Comparison between Tabu and Neighborhood Strategies

20 runs of 1h, 31 hardest GCP instances

Compare with Wilcoxon signed-rank, p-value < 0.001

- TabuCol's best tabu strategy is the Tabu Matrix
- TabuEdges's best tabu strategies are:
  - Tabu List (2 new best scores)
  - Tabu Matrix (1 new best score)
- TabuEdges better than TabuCol on more instances (12 againt 8) and reach more best scores
- TabuDouble (alternate between the two):
  better than TabuCol but not TabuEdges

Comparison with State of the Art

k-TabuCol, k-HEAD and k-AHEAD solve *k*-coloring (1h max to solve 1 color)

/31 instances	# BKS	# Best Score	# Best Mean
TabuCol TL	4	7	5
TabuCol TM	5	8	6
TabuCol CC	0	0	0
TabuEdge TL	14(**2**)	17	**15**
TabuEdge TM	**15**(1)	18	11
TabuEdge CC	14	15	12
TabuDouble TL	14	15	10
TabuDouble TM	13	14	10
k-TabuCol	5	8	7
k-HEAD	7	17	14
k-AHEAD	11	**19**	12

TL : TabuList, TM : TabuMatrix, CC: Configuration Checking

BKS : Best Known Score

Results - State of the Art

instance	BKS	TabuCol TM		TabuEdge TL		TabuEdge TM		TabuDouble TL		k-TabuCol		k-HEAD		k-AHEAD	
		best	time	best	time	best	time	best	time	best	time	best	time	best	time
C2000.5	145	164	2445	166	3290	166	3167	165	663	162	2564	**150**	3300	151	3273
C2000.9	404	413	3491	423	3418	421	3184	422	1525	412	2225	**406**	2527	**406**	3139
C4000.5	259	306	2821	319	3410	317	3499	307	540	304	2332	**283**	3492	284	3156
DSJC500.1	12	**12**	813	13	0	13	0	**12**	1449	**12**	58	**12**	107	**12**	103
DSJC500.5	47	49	239	50	430	50	751	49	2594	49	573	**48**	1239	**48**	1663
DSJC500.9	126	**126**	1033	127	149	**126**	819	127	2249	**126**	1975	**126**	1188	**126**	530
DSJC1000.1	20	21	2	21	193	21	366	21	16	21	0	21	1	**20**	2496
DSJC1000.5	82	88	2919	90	1362	90	2318	90	1315	88	1544	**83**	2811	**83**	2793
DSJC1000.9	222	224	1870	224	1894	**223**	2945	227	409	224	3296	**223**	2272	**223**	3079
flat300_28_0	28*	**28**	1608	**28**	2099	**28**	2584	**28**	620	30	1765	30	1605	30	1228
flat1000_50_0	50*	**50**	920	**50**	16	**50**	25	**50**	15	**50**	27	**50**	69	**50**	18
flat1000_60_0	60*	**60**	2845	**60**	1403	**60**	1327	**60**	1468	**60**	41	**60**	112	**60**	57
flat1000_76_0	76*	87	1994	89	2229	89	1728	89	1413	86	2678	**82**	3064	83	2280
latin_square_10	97	101	2692	101	1014	101	1238	101	3213	100	1014	101	2591	**99**	2859
le450_25c	25*	26	0	26	0	26	0	26	0	26	0	26	0	**25**	1328
le450_25d	25*	26	0	26	0	26	0	26	0	26	0	26	0	**25**	1655
DSJR500.5	122*	125	842	**122**	1	**122**	0	**122**	1	126	231	124	1869	124	1498
r250.5	65*	66	1095	**65**	0	**65**	0	**65**	0	67	129	**65**	2816	**65**	1556
r1000.1c	98	115	254	**98**	21	**98**	9	**98**	24	133	0	100	2306	101	1699
r1000.5	234	245	2383	**234**	49	**234**	41	**234**	44	245	1387	246	2545	244	1862
wap01a	41*	42	62	**41**	7	**41**	4	**41**	7	42	1069	42	219	**41**	2774
wap02a	40*	41	1462	**40**	7	**40**	4	**40**	7	41	231	41	26	**40**	2844
wap03a	43	47	2380	**42**	2143	**42**	2811	**43**	220	45	332	45	510	44	2008
wap04a	41	43	2036	**41**	2235	**41**	1486	42	138	42	755	43	1736	43	612
wap06a	40*	41	778	**40**	1	**40**	0	**40**	0	**40**	1616	**40**	1103	**40**	558
wap07a	41	42	1634	**40**	1296	**41**	19	**41**	27	42	442	42	1667	42	968
wap08a	40*	41	1046	**40**	130	**40**	150	**40**	68	41	915	42	107	41	1568
#BKS		5		14		15		14		5		7		11	
#Best		8		17		18		15		8		17		19	
#Best Avg		6		15		11		10		7		14		12	

Results - Heterogeneous vs Homogeneous Degree

instance	BKS	TabuCol TM		TabuEdge TL		TabuEdge TM		TabuDouble TL		k-TabuCol		k-HEAD		k-AHEAD	
		best	time	best	time	best	time	best	time	best	time	best	time	best	time
C2000.5	145	164	2445	166	3290	166	3167	165	663	162	2564	**150**	3300	151	3273
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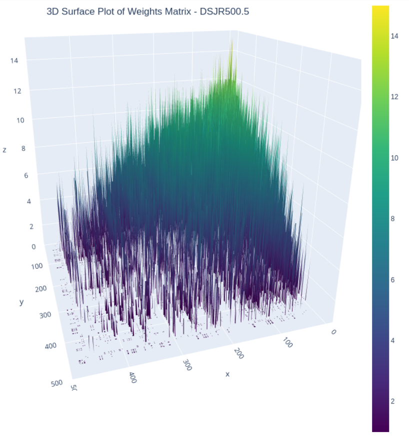
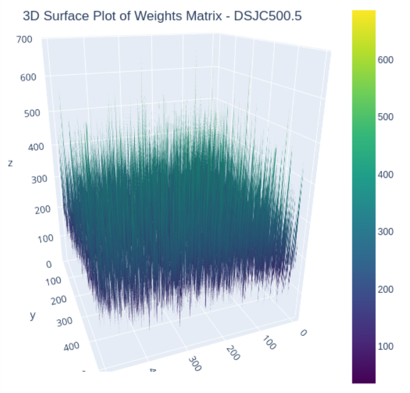
#BKS		5		14		15		14		5		7		11	
#Best		8		17		18		15		8		17		19	
#Best Avg		6		15		11		10		7		14		12	

link DSJR500.5

link DSJC500.5





Conclusion

Conclusion - TabuEdges

Cons

- Pretty bad results on graphs with homogeneous degree
- Doesn't work well on k-coloring

Pros

- Quite fast and very efficient on graphs with heterogeneous degree
- Works well on GCP

Other

- 2 new best scores (wap03a, wap07a)
- Optimal score for wap07a thanks to lower bound from Heule *et al.* [2022]

What's next?

Elements to explore

- Reduce weights or reset weights
- **Guided Memetic Algorithm**
- **aspiration criteria** for the weighted moves
- other **tabu strategies**
- impact of a **swap move**
- go toward an **iterated local search framework** where the weights on edges are used only during a perturbation phase

Thank you for your attention !

Questions ?

Source code, results tables, articles :

`https://cyril-grelier.github.io/`

References

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Comparison between Tabu and Neighborhood Strategies

1 point/instance if the average is significantly better for the method on the line compared to the one in the column (Wilcoxon signed-rank, p-value < 0.001)

/31	TC L	TC M	TC CC	TE L	TE M	TE CC	TD L	TD M	# BKS	# Score	# Mean
TC L	-	3	**21**	5	6	7	5	6	4	15	9
TC M	**15**	-	**31**	8	8	9	6	9	5	16	14
TC CC	0	0	-	0	0	0	0	0	0	0	0
TE L	**14**	**13**	**31**	-	0	**7**	**7**	**5**	14(**2**)	21	**19**
TE M	**14**	**12**	**31**	**2**	-	**7**	**6**	**3**	**15**(1)	**22**	14
TE CC	**13**	**11**	**31**	1	1	-	4	1	14	19	14
TD L	**12**	**11**	**31**	2	2	**7**	-	2	14	20	13
TD M	**13**	**11**	**31**	1	1	**4**	**3**	-	13	19	13

TC : TabuCol, TE : TabuEdges, TD : TabuDouble

L : TabuList, M : TabuMatrix, CC: Configuration Checking

BKS : Best Known Score

Comparison with State of the Art

1 point/instance if the average is significantly better for the method on the line compared to the one in the column (Wilcoxon signed-rank, p-value < 0.001)

/31	TC M	TE L	TE M	TD	k-TC	k-HEAD	k-AHEAD	# BKS	# Score	# Mean
TC M	-	8	8	6	0	1	0	5	8	6
TE L	**13**	-	0	**7**	**11**	**11**	**10**	14(**2**)	17	**15**
TE M	**12**	**2**	-	**6**	**11**	**11**	**10**	**15**(1)	**18**	11
TD	**11**	2	2	-	**11**	**11**	10	14	15	10
k-TC	**10**	8	8	10	-	1	0	5	8	7
k-HEAD	**15**	8	8	9	**12**	-	1	7	17	14
k-AHEAD	**18**	9	9	10	**14**	1	-	11	19	12

TC : TabuCol, TE : TabuEdges, TD : TabuDouble

L : TabuList, M : TabuMatrix, CC: Configuration Checking

BKS : Best Known Score

k- : solve k-coloring problem, (GCP otherwise)