

On Monte Carlo Tree Search for Weighted Vertex Coloring Problem

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Cyril Grelier Olivier Goudet Jin-Kao Hao

LERIA, Université d'Angers, 2 Boulevard Lavoisier, 49045 Angers, France
{cyril.grelier,olivier.goudet,jin-kao.hao}@univ-angers.fr



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- 1 Introduction
 - Problem description
 - State of the art
- 2 MCTS
- 3 Experimentation

WVCP - Weighted Vertex Coloring Problem

Given a graph $G = (V, E)$, V the vertices, E the edges of the graph and $w(v)$ the weight of v for $v \in V$

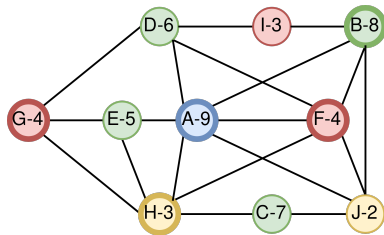
The objective is to find a legal coloring s (two connected vertices can't share the same color) that minimizes the sum of the heaviest vertices of each k colors.

$$\text{score} = F(s) = \sum_{i=1}^k \max_{v \in V_i} w(v)$$

Applications :

- Scheduling on a Batch Machine with Job Compatibilities
- Traffic assignment in communication satellites
- Matrix Decomposition Problem

NP-hard problem



$$\text{Score} = 9 + 8 + 4 + 3 = 24$$

- **R-GRASP** : reactive GRASP, iterated greedy algorithm with local search.
Prais, M., Ribeiro, C.C. , 2000.
Reactive GRASP : An Application to a Matrix Decomposition Problem in TDMA Traffic Assignment.
- **2-Phase** : Phase 1 : generation of independent sets, phase 2 : optimization.
Malaguti, E., Monaci, M., Toth, P., 2009.
Models and heuristic algorithms for a weighted vertex coloring problem.
- **MWSS** : mixed integer linear programming.
Cornaz, D., Furini, F., Malaguti, E., 2017.
Solving vertex coloring problems as maximum weight stable set problems.
- **AFISA** : tabu search with coefficient to manage GCP and WVCP.
Sun, W., Hao, J.-K., Lai, X., Wu, Q., 2018.
Adaptive feasible and infeasible tabu search for weighted vertex coloring.
- **RedLS** : reduction and local search with weight management of the edges.
Wang, Y., Cai, S., Pan, S., Li, X., Yin, M., 2020.
Reduction and Local Search for Weighted Graph Coloring Problem.
- **ILS-TS** : reduction and iterated local search with grenade operator.
Nogueira, B., Tavares, E., Maciel, P., 2021.
Iterated local search with tabu search for the weighted vertex coloring problem.
- **DLMCOL** : memetic algorithm with deep learning for the crossover selection.
Goudet, O., Grelier, C., Hao, J.-K., 2021.
A deep learning guided memetic framework for graph coloring problems.

1 Introduction

2 MCTS

- Why an MCTS ?
- Proposed algorithm
- Pre-processing : Graph reduction
- Beginning of turn 6
- Selection
- Expansion
- Simulation
- Update
- Pruning

3 Experimentation

MCTS – Monte Carlo Tree Search

What are the particularities of this coloring problem ?

- Some vertices are more important than others (weight, degree)
- Only the vertex of maximum weight of each color group has an influence on the score (many plateaus in the fitness landscape)

Why an MCTS ?

- MCTS learns to find a good color for the heaviest vertices
- MCTS continuously explores new area in the search space
- The way of exploring the tree cut most of the color symmetries
- If the tree is completely explored (with pruning) then the solution is optimal
- Can be combined with local search or other algorithm

- Pre-processing :
 - ① Graph reduction with 2 rules ([Wang et al. 2020,Cheeseman et al. 1991])
 - ② Vertices sorted by weight (heavier first then degree, higher first)
- While stop criteria (time, iterations, tree fully explored (optimal solution)) :
 - ① MCTS 1 phase : Selection
Score computation and selection adapted for optimisation problem ([Jookan et al. 2020])

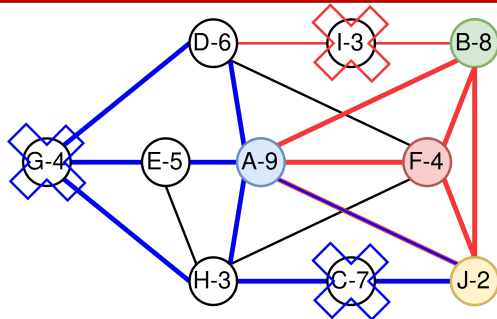
$$normalizedScore(C_{t+1}^i) = \frac{rank(C_{t+1}^i)}{\sum_{i=1}^I rank(C_{t+1}^i)}$$

Select the child C^i that maximizes :

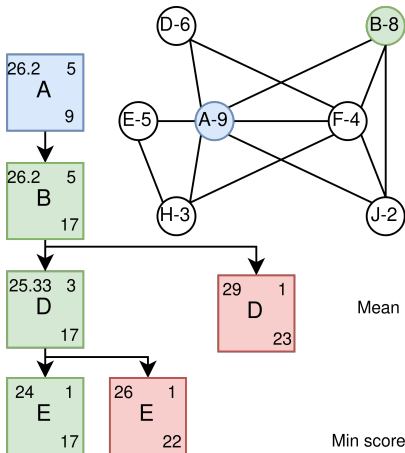
$$score_{UCB}(C^i) = normalizedScore(C_{t+1}^i) + c * \sqrt{\frac{2 * \ln(numberVisits(C^t))}{numberVisits(C_{t+1}^i)}}$$

- ② MCTS 2 phase : Expansion
Opening of the nodes sorted by color
Pruning rule adapted for the problem
- ③ MCTS 3 phase : Simulation
3 greedy with or not random choice of the color
4 local search (with different neighborhood operators or perturbations)
- ④ MCTS 4 phase : Update

I weight is less than the weight of the 3rd heaviest vertex of any clique of the graph
 (3rd as degree of I is 2, vertex degree + 1)

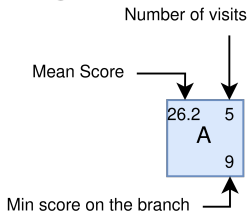


All neighbors of G/C are neighbors of A and G/C less heavy than A



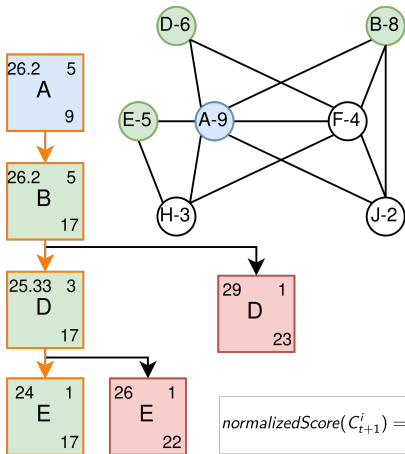
turn 1 : $A + B \rightarrow 26$
 turn 2 : $AB + D \rightarrow 26$
 turn 3 : $AB + D \rightarrow 29$
 turn 4 : $ABD + E \rightarrow 24$
 turn 5 : $ABD + E \rightarrow 26$
 turn 6 :

Beginning of turn 6



turn 1 : A + B -> 26
 turn 2 : A B + D -> 26
 turn 3 : A B + D -> 29
 turn 4 : A B D + E -> 24
 turn 5 : A B D + E -> 26
 turn 6 : A B D E

1) Selection



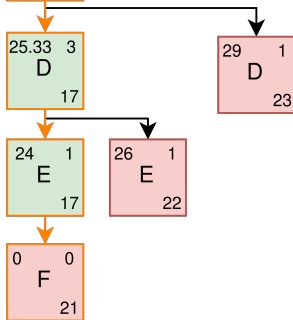
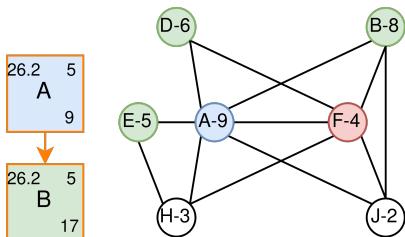
$$\text{normalizedScore}(C_{t+1}^i) = \frac{\text{rank}(C_{t+1}^i)}{\sum_{i=1}^I \text{rank}(C_{t+1}^i)}$$

Select the child that maximizes :

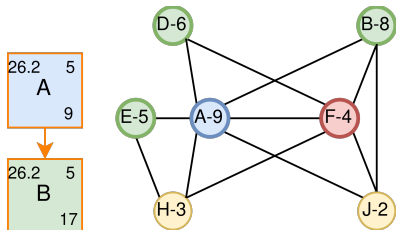
$$\text{score}_{UCB}(C^i) = \text{normalizedScore}(C_{t+1}^i) + c * \sqrt{\frac{2 * \ln(\text{numberVisits}(C^t))}{\text{numberVisits}(C_{t+1}^i)}}$$

turn 1 : $A + B \rightarrow 26$
 turn 2 : $AB + D \rightarrow 26$
 turn 3 : $AB + D \rightarrow 29$
 turn 4 : $ABD + E \rightarrow 24$
 turn 5 : $ABD + E \rightarrow 26$
 turn 6 : $ABCDE + F$

2) Expansion

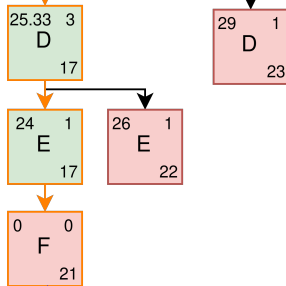


Opening the nodes sorted by color



turn 1 : $A + B \rightarrow 26$
 turn 2 : $AB + D \rightarrow 26$
 turn 3 : $AB + D \rightarrow 29$
 turn 4 : $ABD + E \rightarrow 24$
 turn 5 : $ABD + E \rightarrow 26$
 turn 6 : $ABCDE + F \rightarrow 24$

3) Simulation

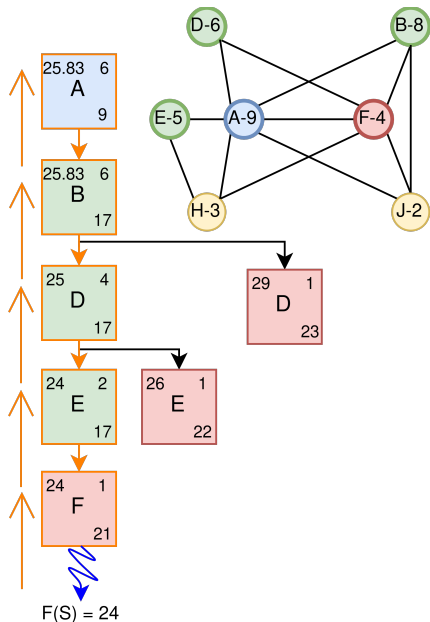


3 greedy (-/+ random)

4 local search :

- AFISA
- TabuWeight (equivalent of TabuCol)
- RedLS
- ILSTS

$F(S) = 24$



turn 1 : $A + B \rightarrow 26$

turn 2 : $AB + D \rightarrow 26$

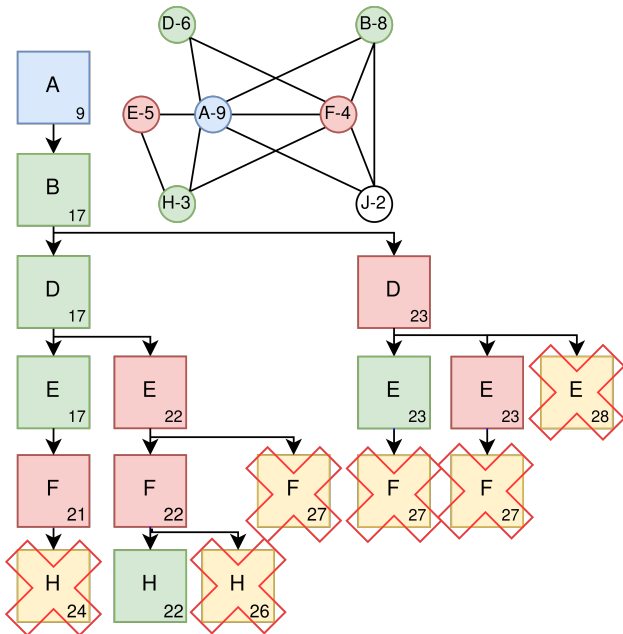
turn 3 : $AB + D \rightarrow 29$

turn 4 : $ABD + E \rightarrow 24$

turn 5 : $ABD + E \rightarrow 26$

turn 6 : $ABDE + F \rightarrow 24$

4) Update

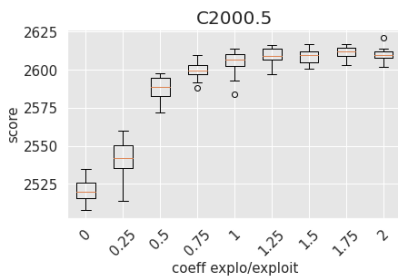
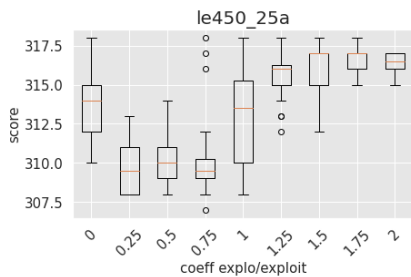
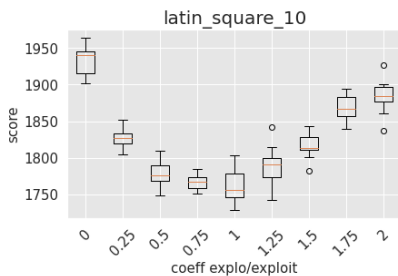
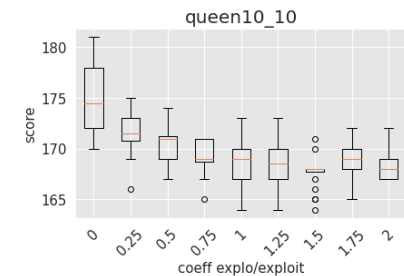


1 Introduction

2 MCTS

3 Experimentation

- Analysis of coefficient exploration vs exploitation
- Comparison with greedy and local search



Tested on the 161 instances of the state of the art

https://github.com/Cyril-Grelier/gc_wvcp_mcts for complete data tables

	greedy		mcts + greedy		
	nb best	significantly better	nb best	significantly better	nb optimal
random	1	0	56	161	33
constrained	18	0	95	157	33
deterministic	18	0	72	140	34

	local search		mcts + local search		
	nb best	significantly better	nb best	significantly better	nb optimal
afisa	26	28	100	66	12
tabu weight	74	44	102	54	28
redls	99	35	109	60	28
ilsts	131	6	129	2	15

significantly better with a p-value ≤ 0.001

Conclusion

Discussion

- Exploration vs exploitation ratio can be tricky to define
- Possible optimal solution
- "Smart" restarts allow better exploration of the search space than weak perturbations.
- Can improve weak local search but stronger local search can be difficult to improve with short time out

Thank you for your attention !

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