Monte Carlo Tree Search for Weighted Vertex Coloring Problem [1]

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1. Grelier, C., Goudet, O., Hao, J.-K., 2022. On Monte Carlo Tree Search for Weighted Vertex Coloring. arXiv:2202.01665 [cs].

WVCP - Weighted Vertex Coloring Problem

Given a graph $G = (V, E)$, V the vertices, E the edges of the graph and $w(v)$ the weight of v for $v \in V$

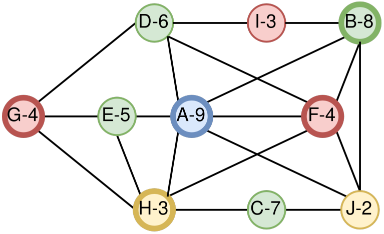
The objective is to find a legal coloring s (two connected vertices can't share the same color) that minimizes the sum of the heaviest vertices of each k colors.

$$score = F(s) = \sum_{i=1}^{k} max_{v \in V_i} w(v)$$

Applications :

- Scheduling on a Batch Machine with Job Compatibilities
- Traffic assignment in communication satellites
- Matrix Decomposition Problem

NP-hard problem



Score = 9 + 8 + 4 + 3 = 24

- R-GRASP [1] (reactive GRASP, iterated greedy algorithm with local search)
- 2-Phase [2] (Phase 1 : generation of independent sets, phase 2 : optimization)
- MWSS [3] (mixed integer linear programming)
- AFISA [4] (tabu search with coefficient to manage GCP and WVCP)
- RedLS [5] (reduction and local search with weight management of the edges)
- ILS-TS [6] (reduction and iterated local search with grenade operator)
- DLMCOL [7] (memetic algorithm with deep learning for the crossover selection)

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1. Prais, M., Ribeiro, C.C., 2000. Reactive GRASP : An Application to a Matrix Decomposition Problem in TDMA Traffic Assignment. INFORMS Journal on Computing 12, 164–176.
2. Malaguti, E., Monaci, M., Toth, P., 2009. Models and heuristic algorithms for a weighted vertex coloring problem. J Heuristics 15, 503–526. https ://doi.org/10.1007/s10732-008-9075-1
3. Cornaz, D., Furini, F., Malaguti, E., 2017. Solving vertex coloring problems as maximum weight stable set problems. Discrete Applied Mathematics 217, 151–162.
4. Sun, W., Hao, J.-K., Lai, X., Wu, Q., 2018. Adaptive feasible and infeasible tabu search for weighted vertex coloring. Information Sciences 466, 203–219.
5. Wang, Y., Cai, S., Pan, S., Li, X., Yin, M., 2020. Reduction and Local Search for Weighted Graph Coloring Problem. AAAI 34, 2433–2441.
6. Nogueira, B., Tavares, E., Maciel, P., 2021. Iterated local search with tabu search for the weighted vertex coloring problem. Computers & Operations Research 125, 105087.
7. Goudet, O., Grelier, C., Hao, J.-K., 2021. A deep learning guided memetic framework for graph coloring problems. arXiv :2109.05948 [cs].

MCTS – Monte Carlo Tree Search

What are the particularities of this coloring problem ?

- Some vertices are more important than others (weight, degree)
- Only the vertex of maximum weight of each color group has an influence on the score (many plateaus in the fitness landscape)

Why an MCTS ?

- MCTS learns to find a good color for the heaviest vertices
- MCTS continuously explores new area in the search space
- The way of exploring the tree cut most of the color symmetries
- If the tree is completely explored (with pruning) then the solution is optimal
- Can be combined with local search or other algorithm

- Pre-processing :
  1. Graph reduction with 2 rules ([Wang et al. 2020,Cheeseman et al. 1991])
  2. Vertices sorted by weight (heavier first then degree, higher first)
- While stop criteria (time, iterations, tree fully explored (optimal solution)) :
  1. MCTS 1 phase : Selection
     Score computation and selection adapted for optimisation problem ([Jooken et al. 2020])

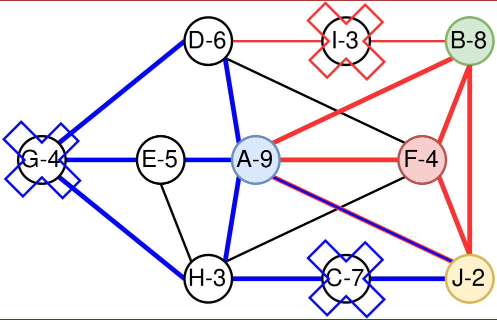
$$normalizedScore(C^i_{t+1}) = \frac{rank(C^i_{t+1})}{\sum_{i=1}^{l} rank(C^i_{t+1})}$$

     Select the child C^i that maximizes :

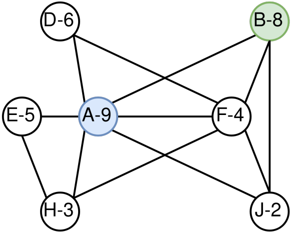
$$score_{UCB}(C^i) = normalizedScore(C^i_{t+1}) + c * \sqrt{\frac{2 * ln(numberVisits(C^t))}{numberVisits(C^i_{t+1})}}$$

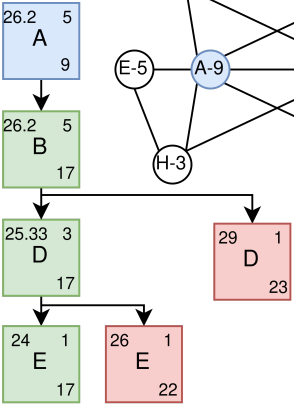
  2. MCTS 2 phase : Expansion
     Opening of the nodes sorted by color
     Pruning rule adapted for the problem
  3. MCTS 3 phase : Simulation
     3 greedy with or not random choice of the color
     4 local search (with different neighborhood operators or perturbations)
  4. MCTS 4 phase : Update

I weight is less than the weight of the 3rd heaviest vertex of any clique of the graph (3rd as degree of I is 2, vertex degree + 1)



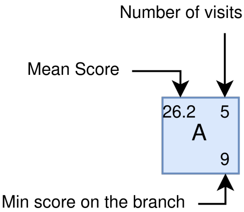
All neighbors of G/C are neighbors of A and G/C less heavy than A

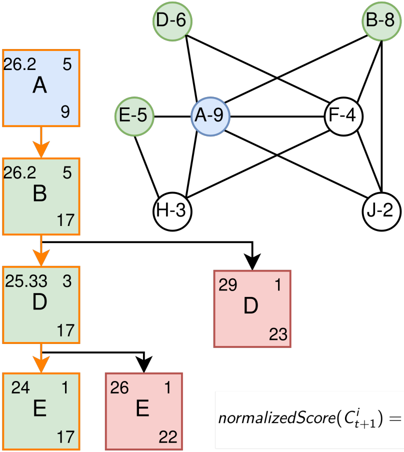




turn 1 : A + B -> 26
turn 2 : A B + D -> 26
turn 3 : A B + D -> 29
turn 4 : A B D + E -> 24
turn 5 : A B D + E -> 26
turn 6 :

Beginning of turn 6





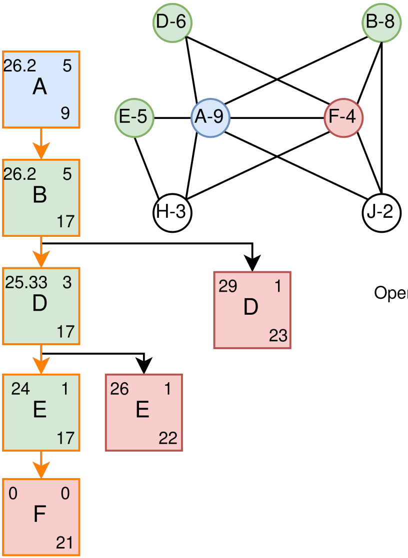
turn 1 : A + B -> 26
turn 2 : A B + D -> 26
turn 3 : A B + D -> 29
turn 4 : A B D + E -> 24
turn 5 : A B D + E -> 26
turn 6 : A B D E

1) Selection

$$normalizedScore(C^i_{t+1}) = \frac{rank(C^i_{t+1})}{\sum_{i=1}^{l} rank(C^i_{t+1})}$$

Select the child that maximizes :

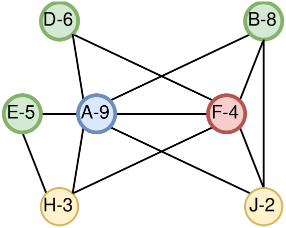
$$score_{UCB}(C^i) = normalizedScore(C^i_{t+1}) + c * \sqrt{\frac{2 * ln(numberVisits(C^t))}{numberVisits(C^i_{t+1})}}$$



turn 1 : A + B -> 26
turn 2 : A B + D -> 26
turn 3 : A B + D -> 29
turn 4 : A B D + E -> 24
turn 5 : A B D + E -> 26
turn 6 : A B D E + F

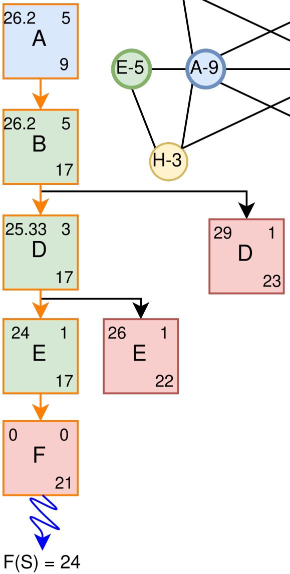
2) Expansion

Opening the nodes sorted by color

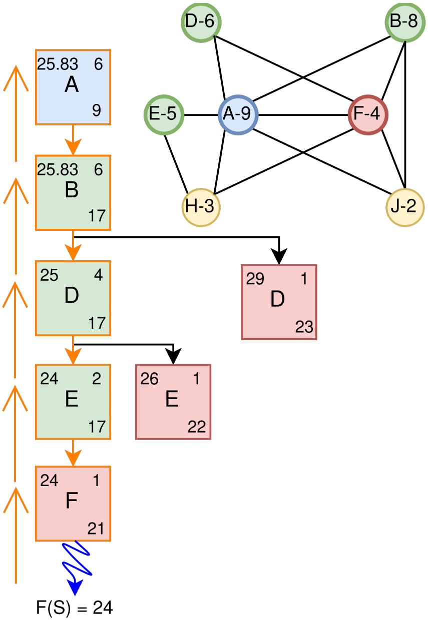


turn 1 : A + B -> 26
turn 2 : A B + D -> 26
turn 3 : A B + D -> 29
turn 4 : A B D + E -> 24
turn 5 : A B D + E -> 26
turn 6 : A B D E + F -> 24

3) Simulation

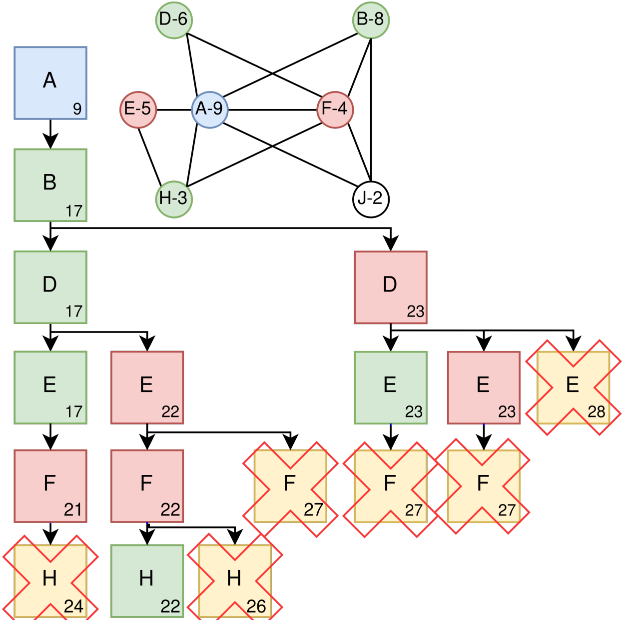


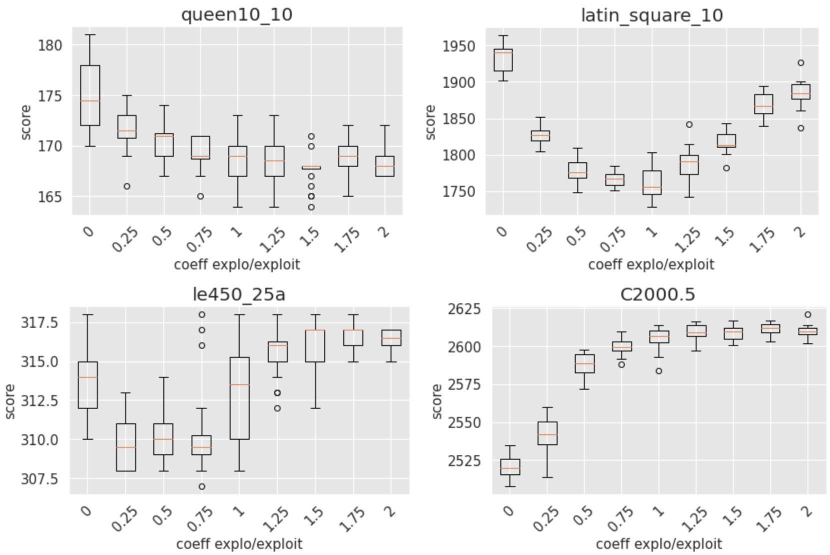
3 greedy (-/+ random)
4 local search :
- AFISA
-TabuWeight (equivalent of TabuCol)
-RedLS
- ILSTS



turn 1 : A + B -> 26
turn 2 : A B + D -> 26
turn 3 : A B + D -> 29
turn 4 : A B D + E -> 24
turn 5 : A B D + E -> 26
turn 6 : A B D E + F -> 24

4) Update


D-6
B-8
E-5
A-9
F-4
H-3
J-2
A 9
B 17
D 17
D 23
E 17
E 22
E 23
E 23
E 28
F 21
F 22
F 27
F 27
F 27
H 24
H 22
H 26


queen10_10
latin_square_10
le450_25a
C2000.5
score
coeff explo/exploit
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
180
175
170
165
1950
1900
1850
1800
1750
317.5
315.0
312.5
310.0
307.5
2625
2600
2575
2550
2525

Tested on the 161 instances of the state of the art
`https://github.com/Cyril-Grelier/gc_wvcp_mcts` for complete data tables

	greedy		mcts + greedy		
	nb best	significantly better	nb best	significantly better	nb optimal
random	1	0	56	161	33
constrained	18	0	95	157	33
deterministic	18	0	72	140	34

	local search		mcts + local search		
	nb best	significantly better	nb best	significantly better	nb optimal
afisa	26	28	100	66	12
tabu weight	74	44	102	54	28
redls	99	35	109	60	28
ilsts	131	6	129	2	15

significantly better with a p-value $<= 0.001$

Conclusion

Discussion

- Exploration vs exploitation ratio can be tricky to define
- Possible optimal solution
- "Smart" restarts allow better exploration of the search space than weak perturbations.
- Can improve weak local search but stronger local search can be difficult to improve with short time out

Thank you for your attention !

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